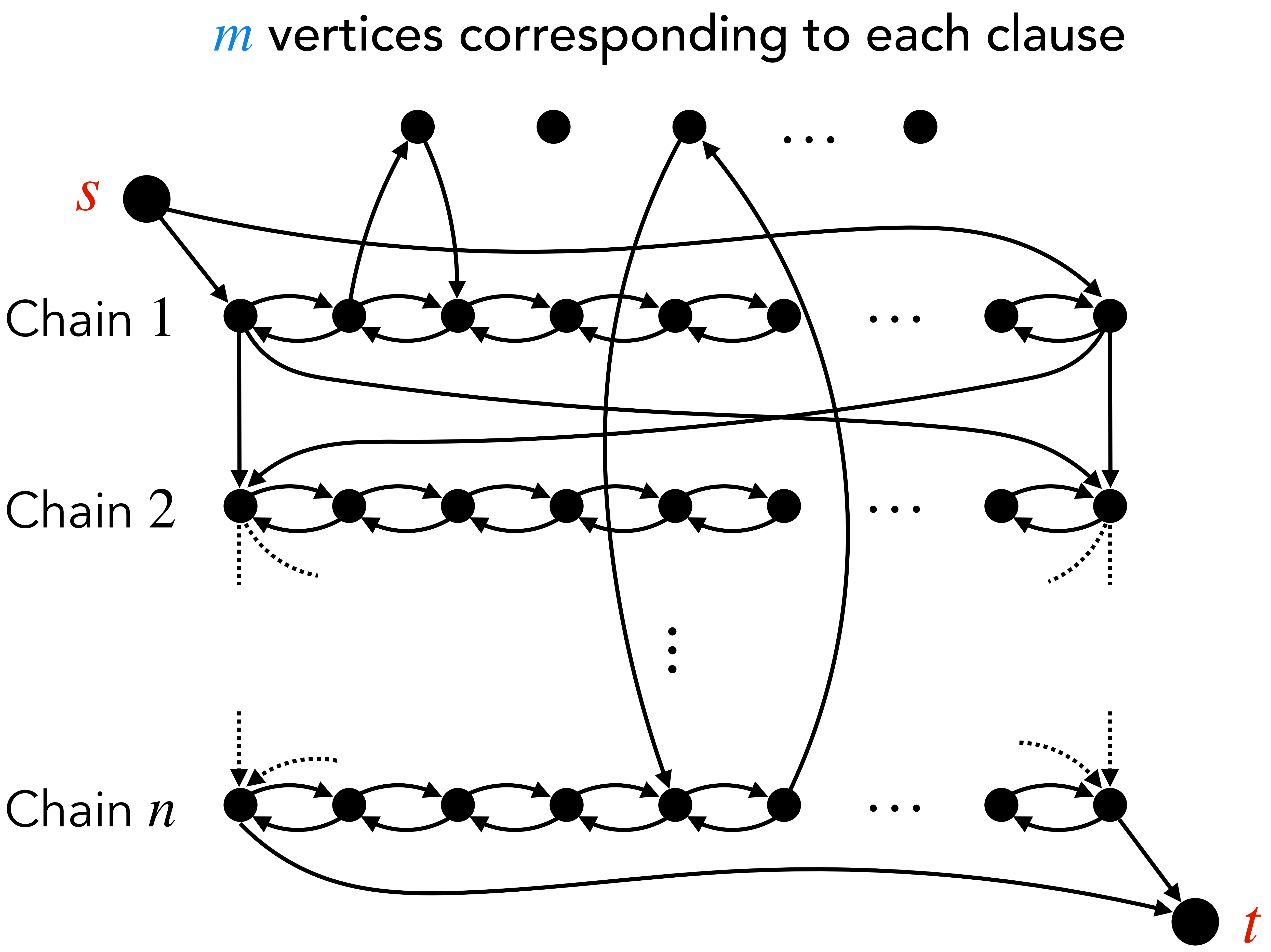
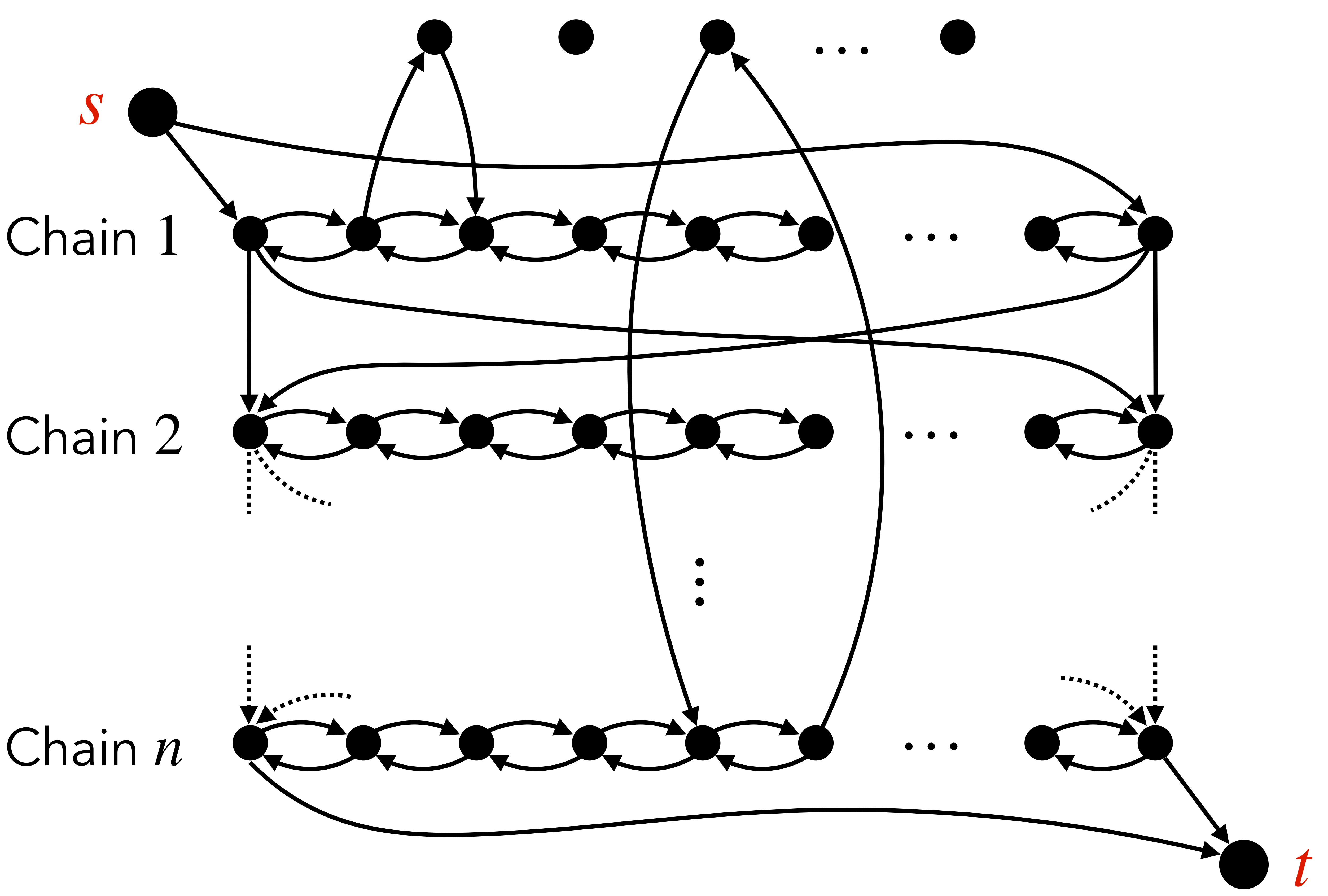
Lecture 37

Reductions: DirHampath (contd.), Hampath, Hamcycle

$3SAT \leq_p$ *DirHampath*

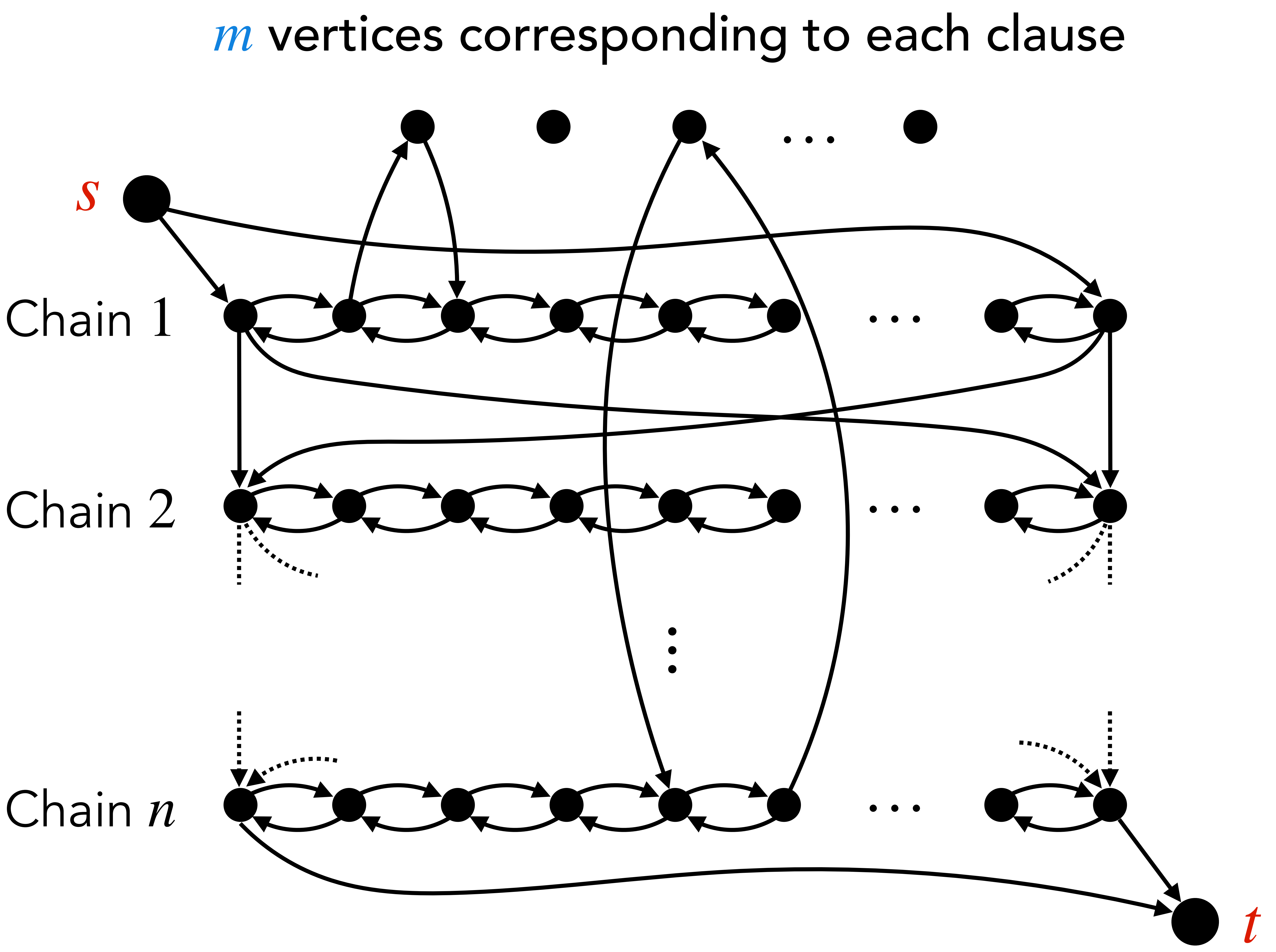


$3SAT \leq_p DirHampath$



Correctness of Reduction ($\Longleftarrow$):

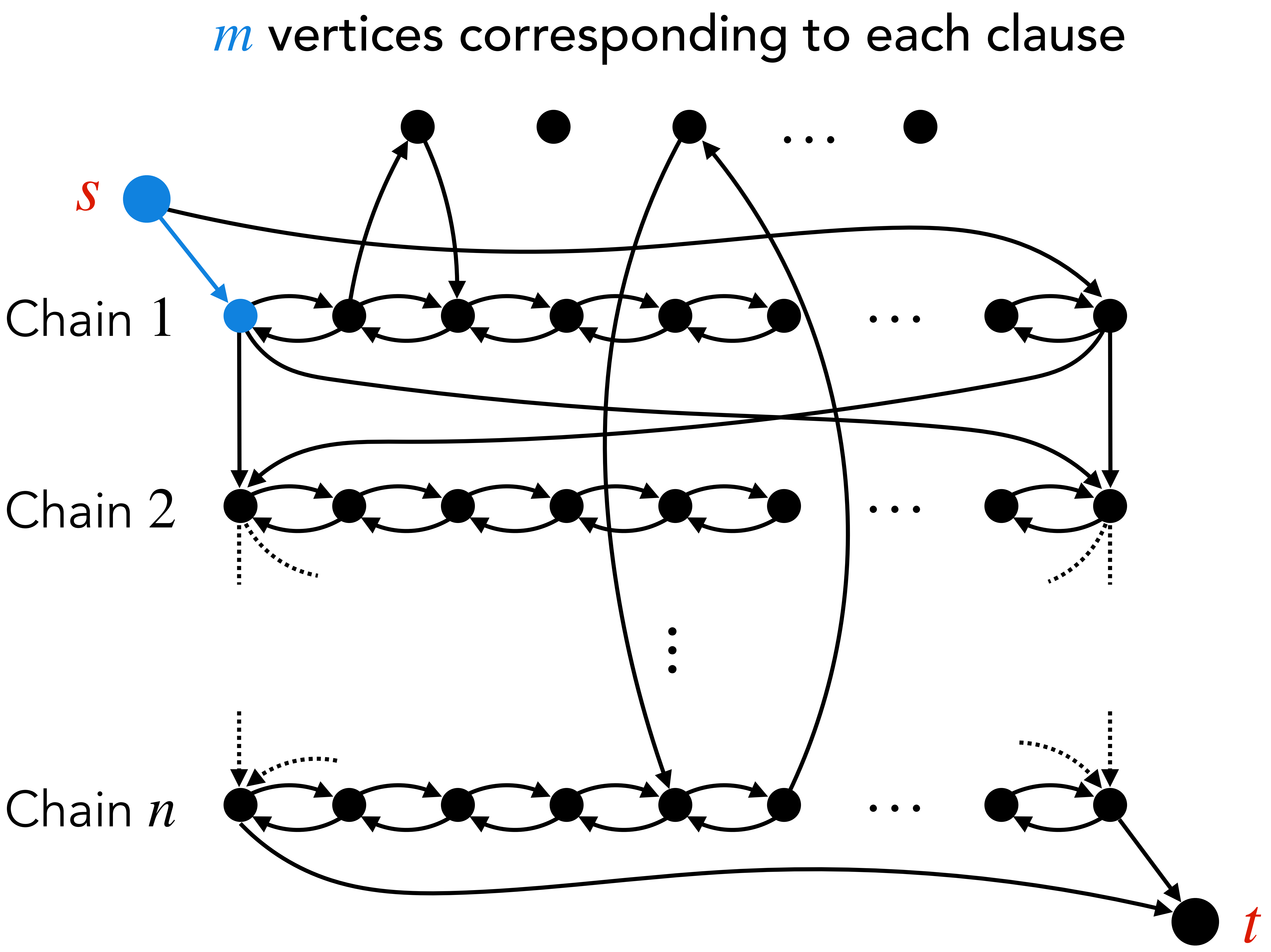
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

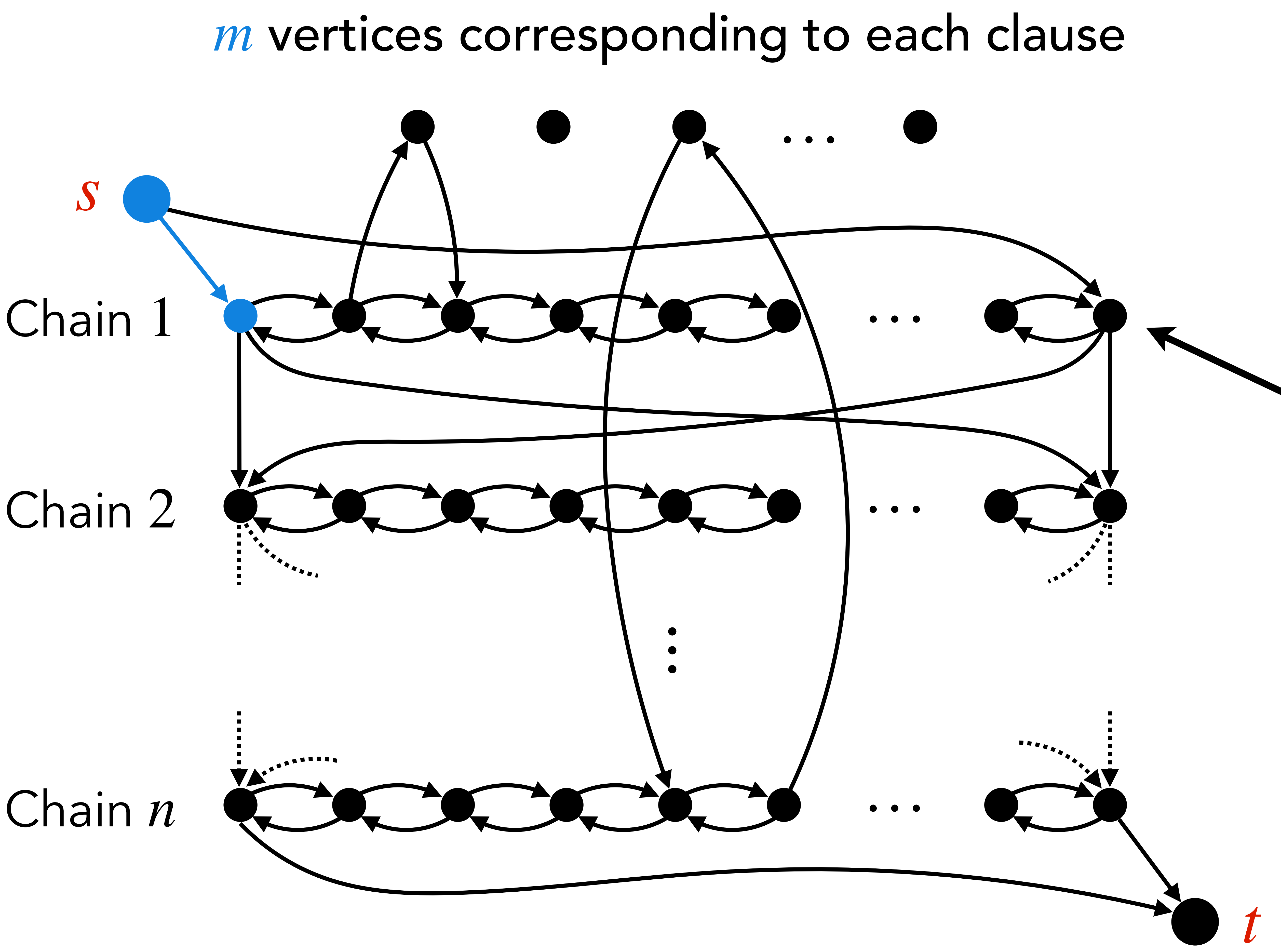
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

3SAT $\leq_p$ DirHampath

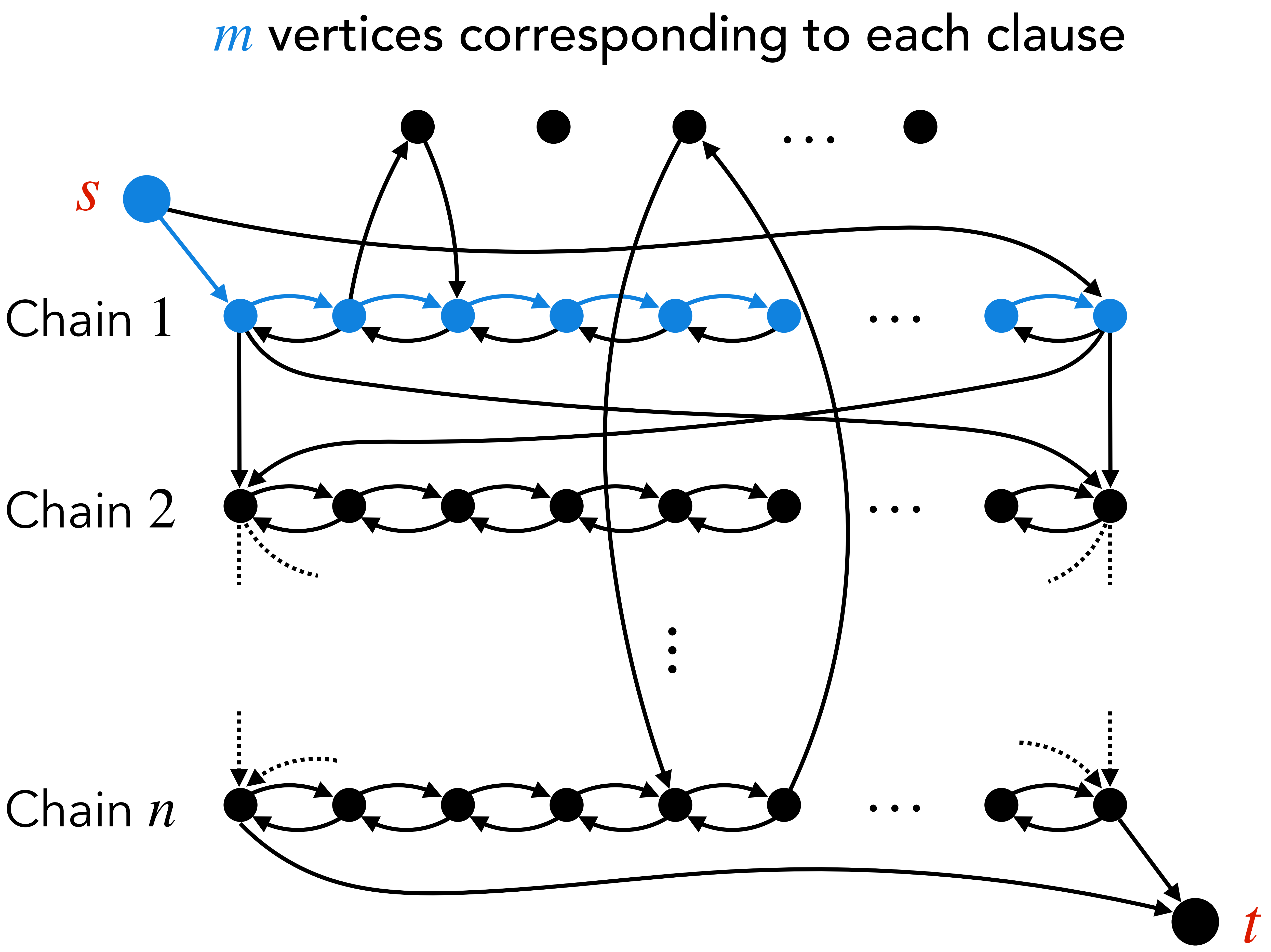


Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

A chain must be visited completely either left-to-right or right-to-left the very first time it is touched in the hamiltonian path.

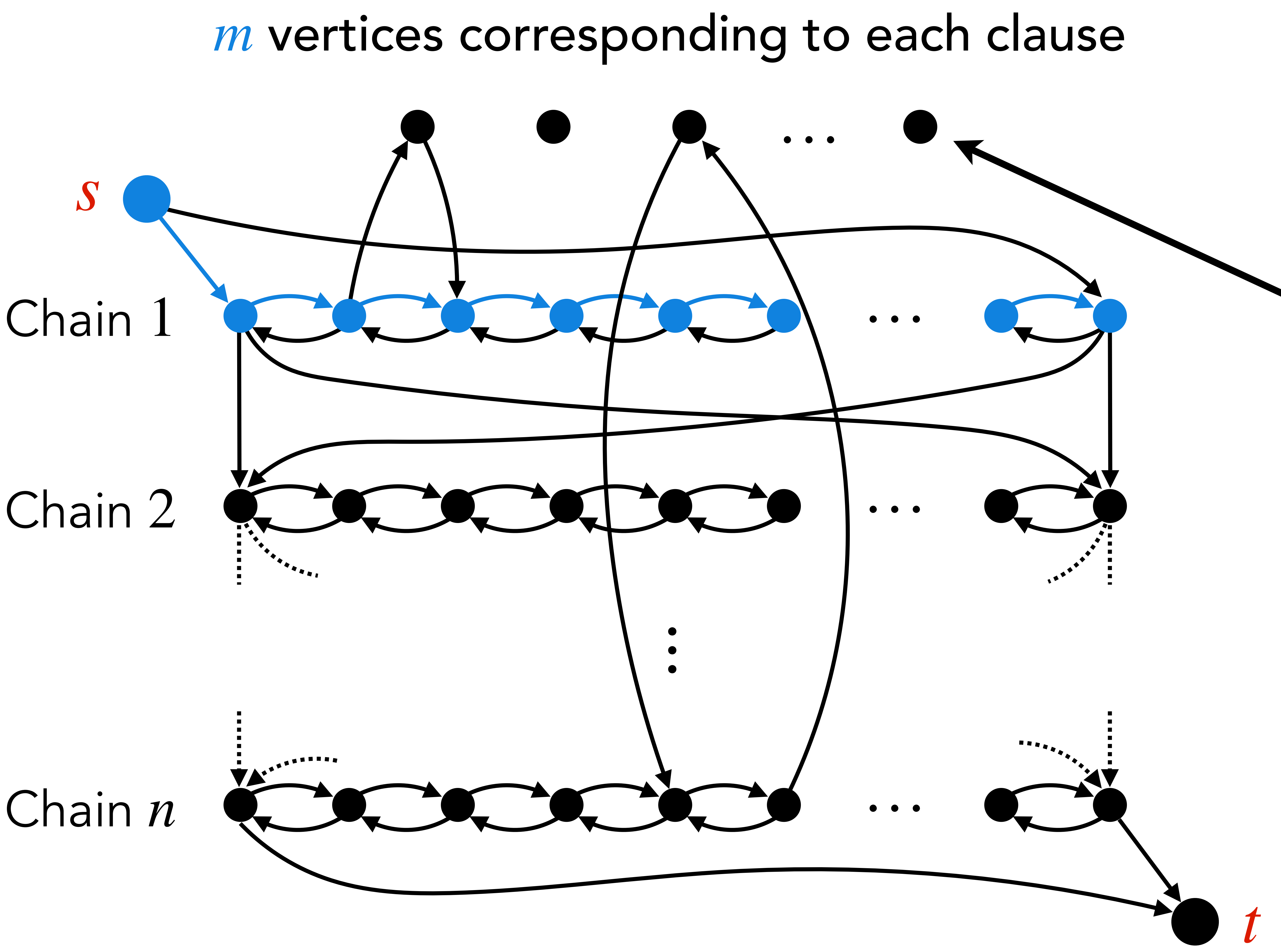
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

$3SAT \leq_p DirHampath$

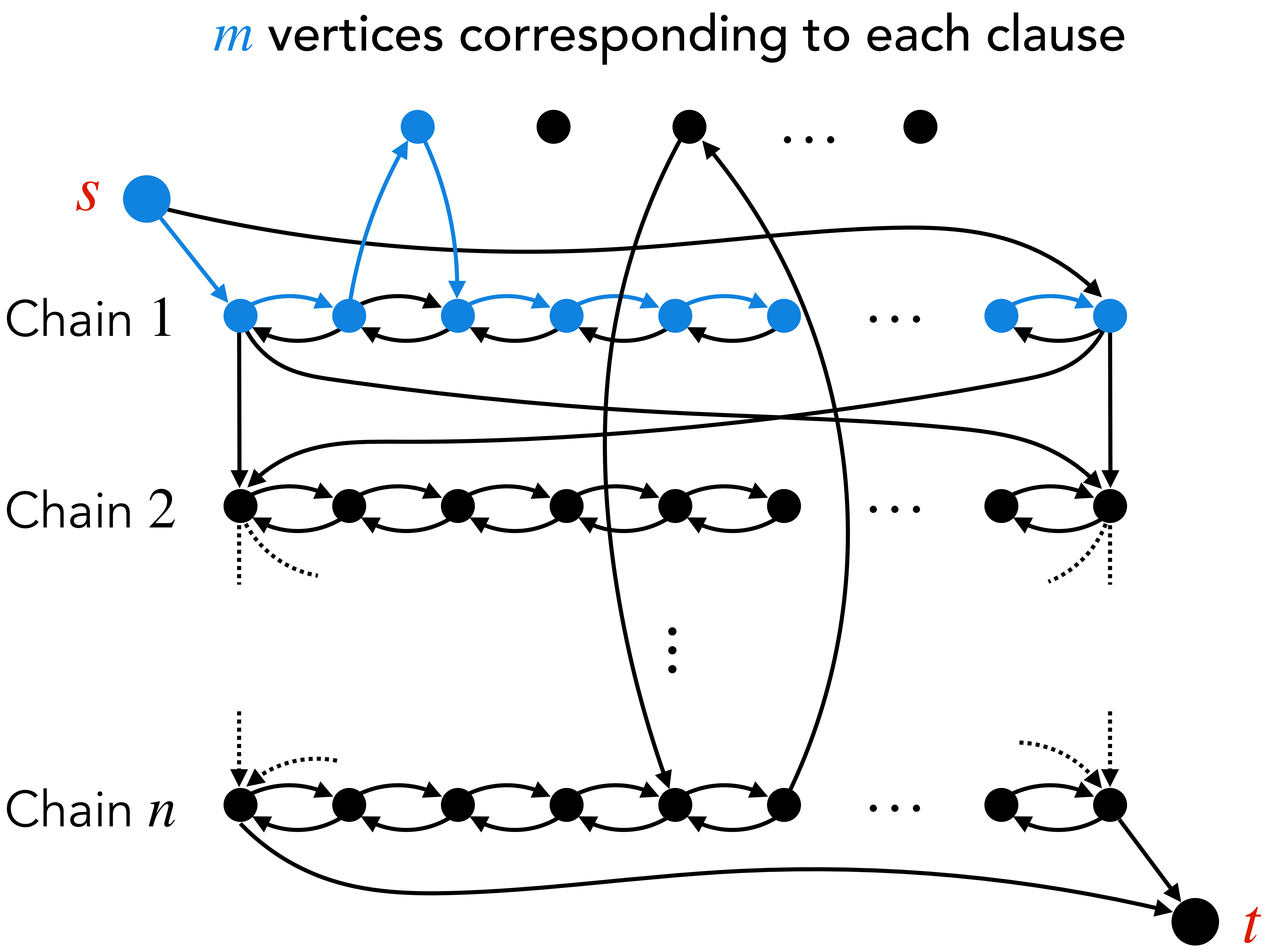


Clause vertices must be visited during chain traversals.

Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian *st*-path.

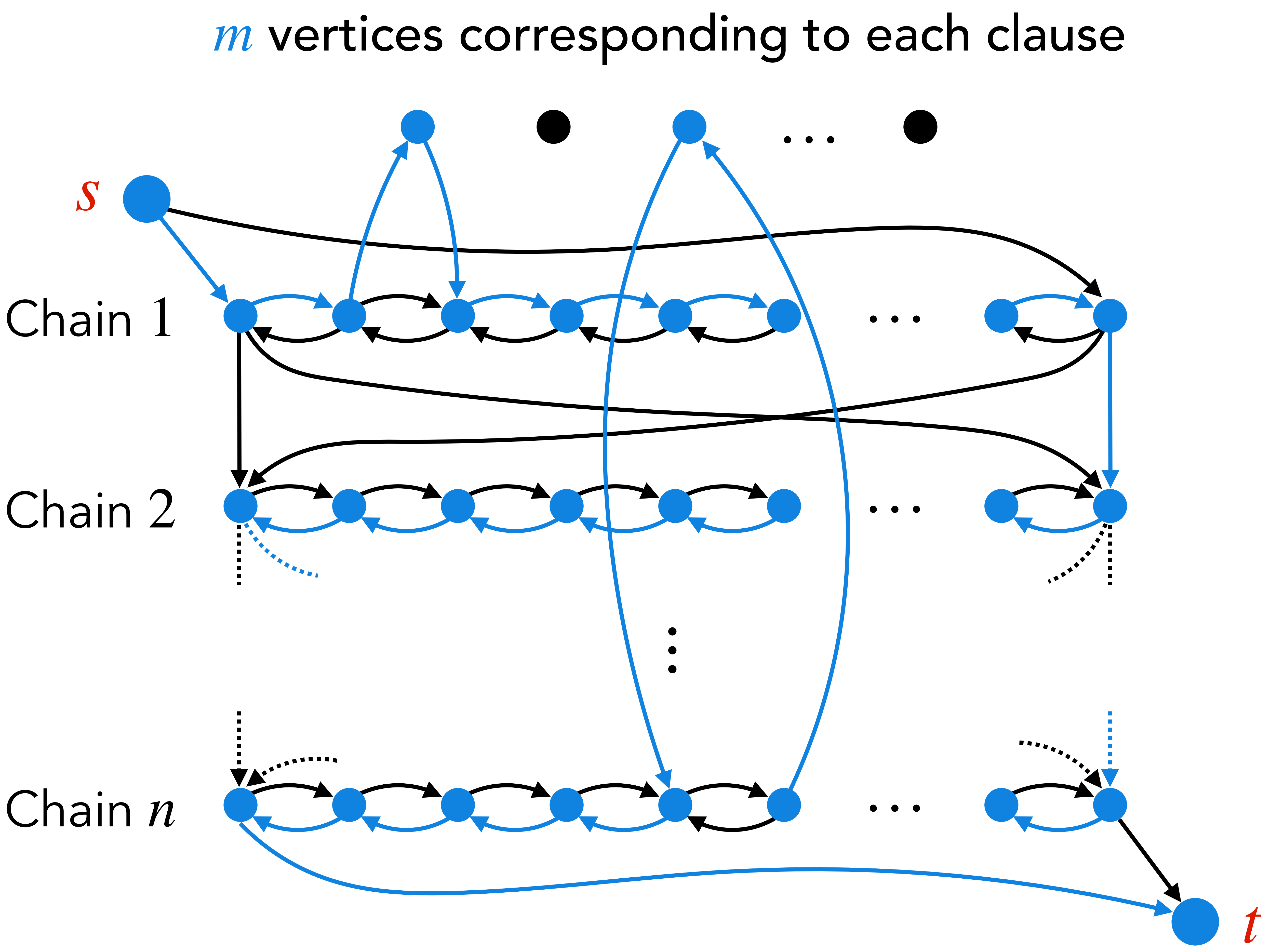
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

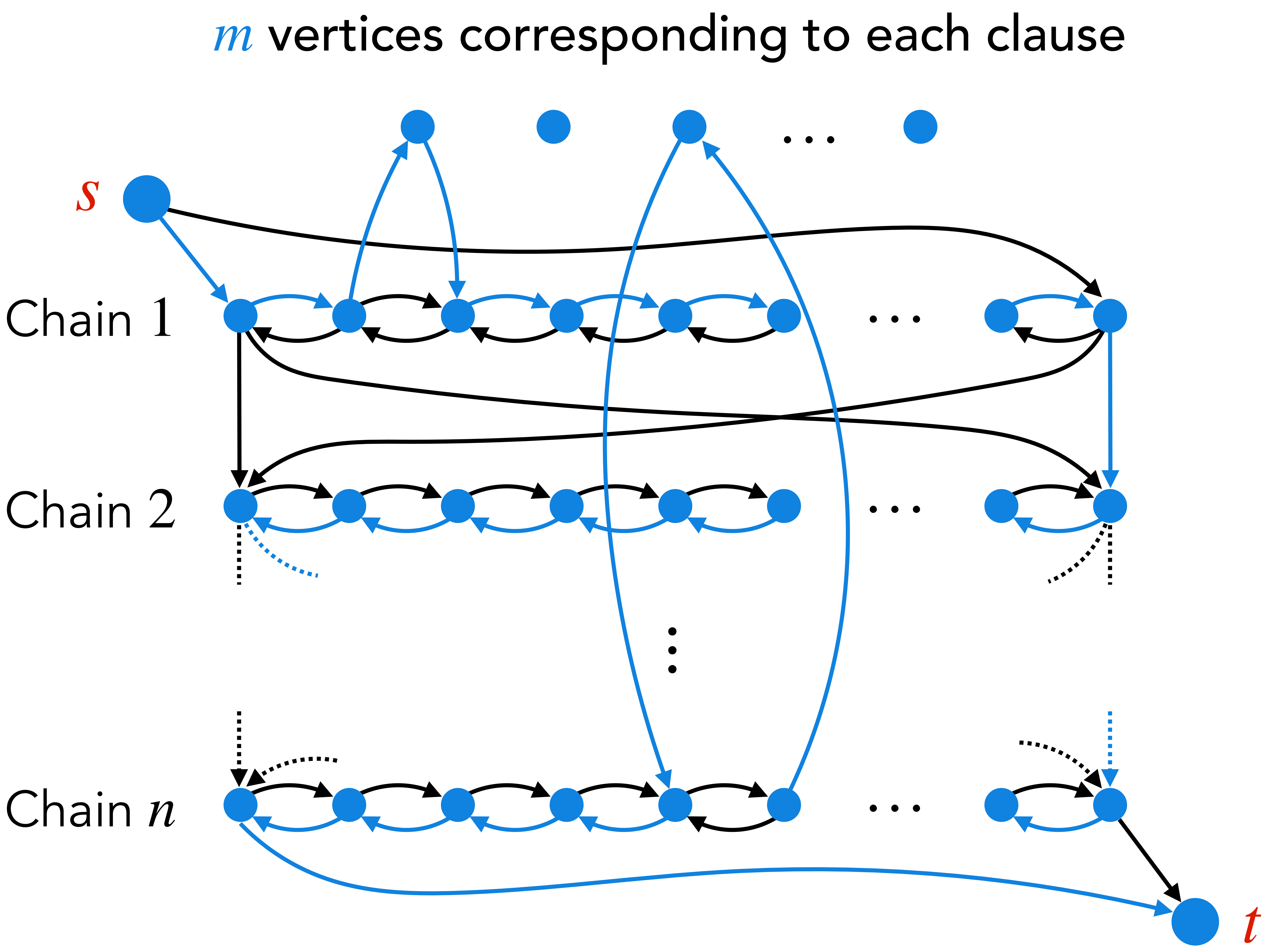
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

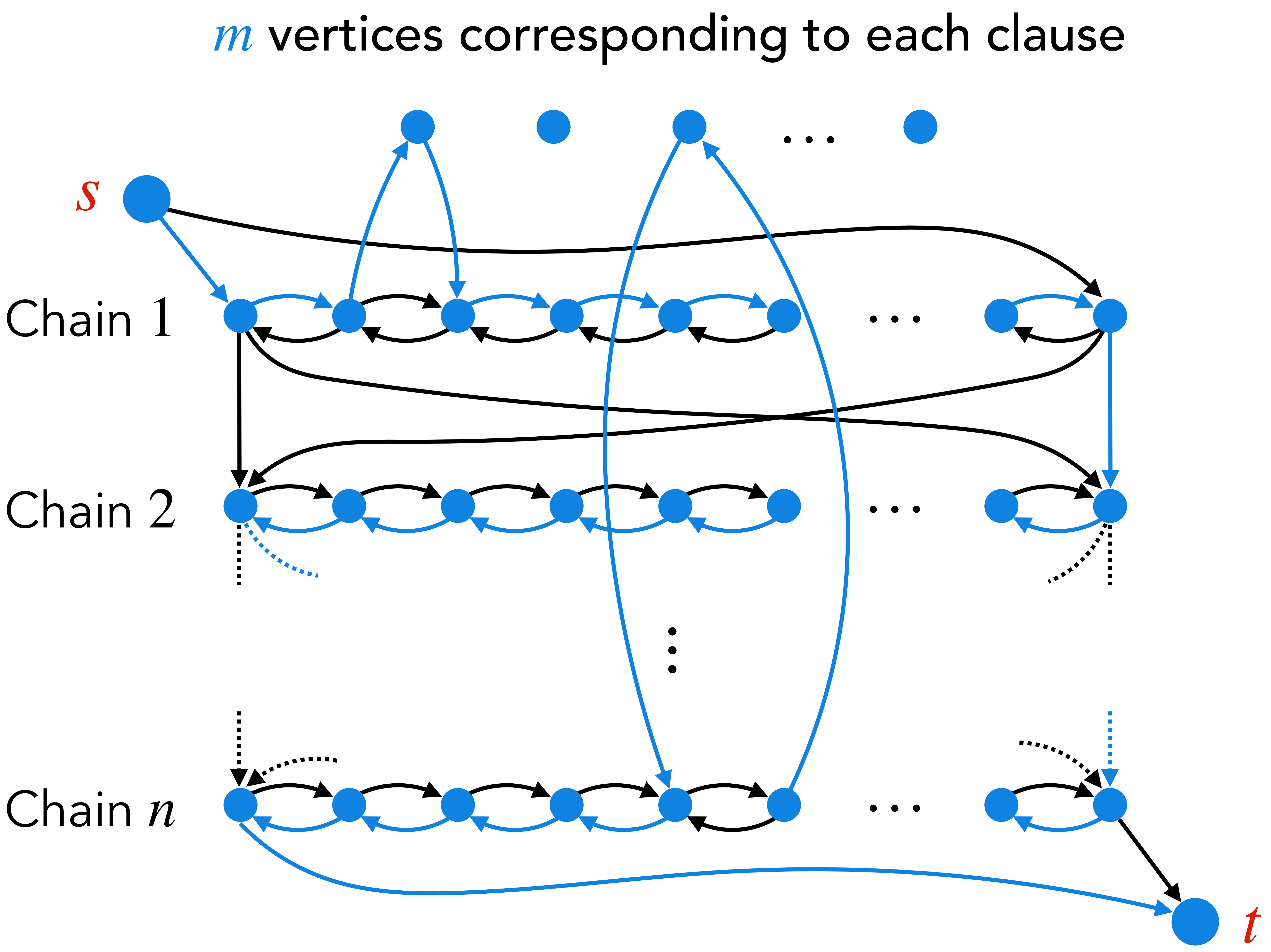
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.

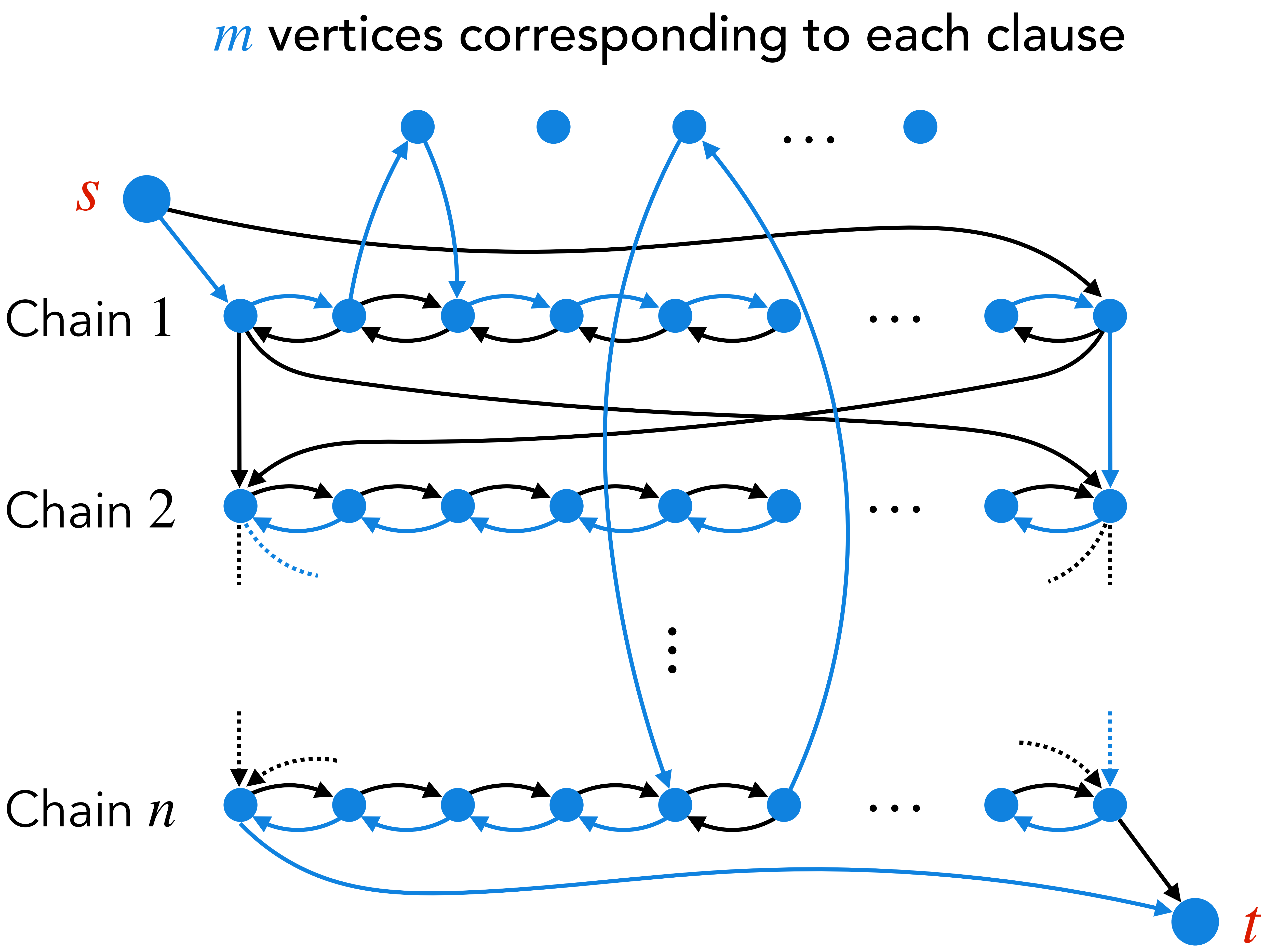
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:

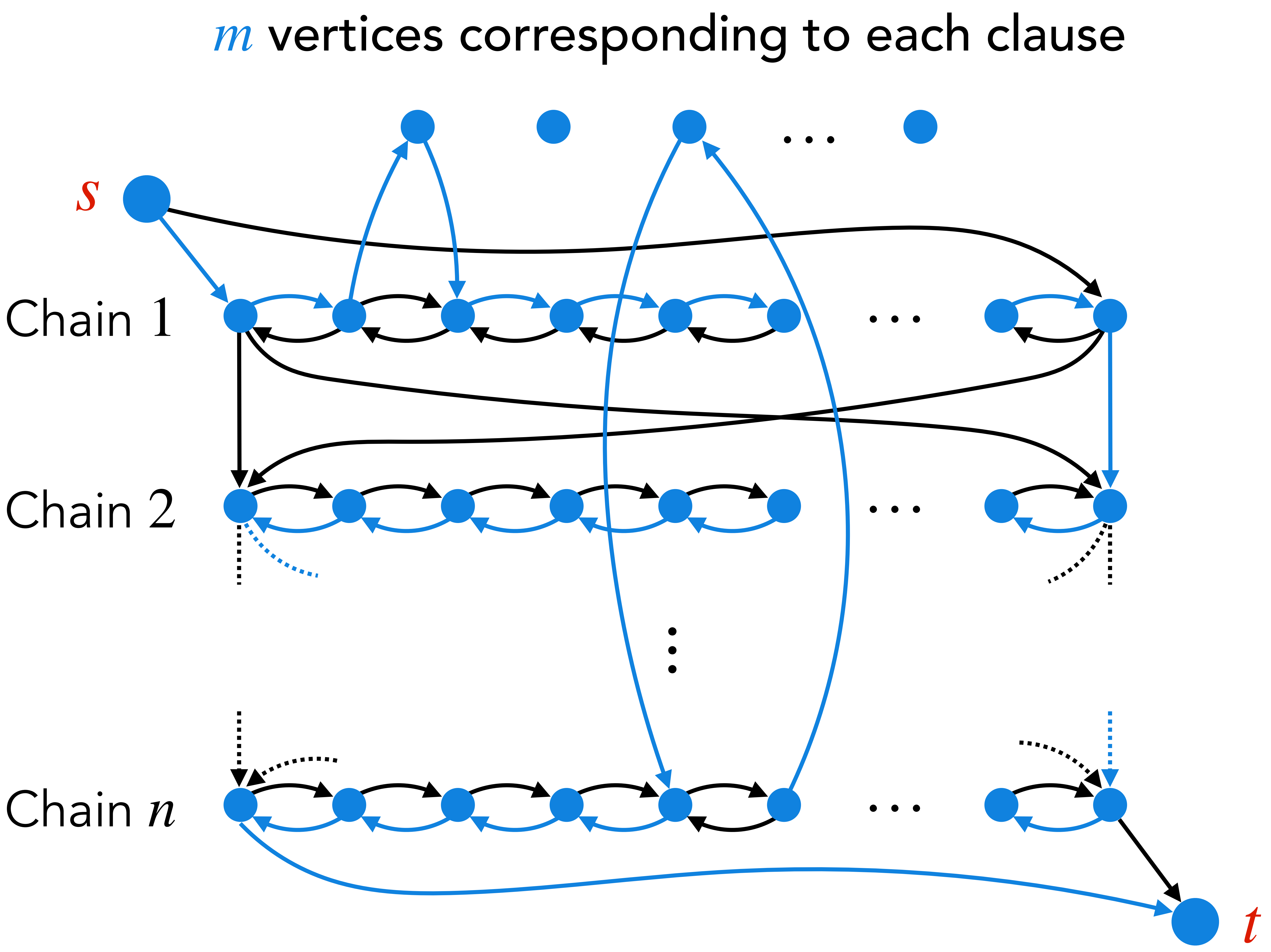
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
    - $u_i = 1$, if ith chain is traversed left-to-right.

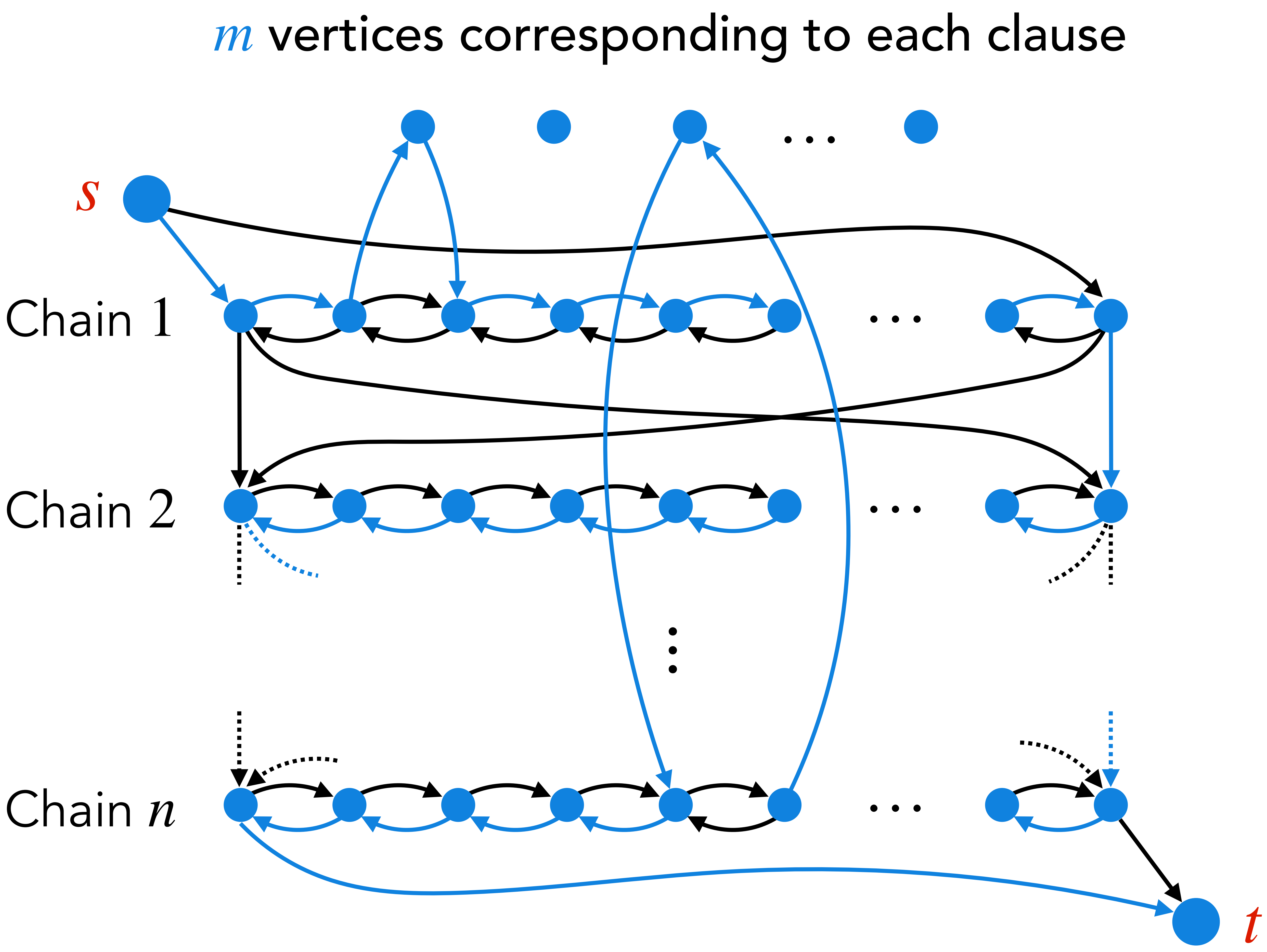
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
  - $u_i = 1$, if ith chain is traversed left-to-right.
  - $u_i = 0$, if ith chain is traversed right-to-left.

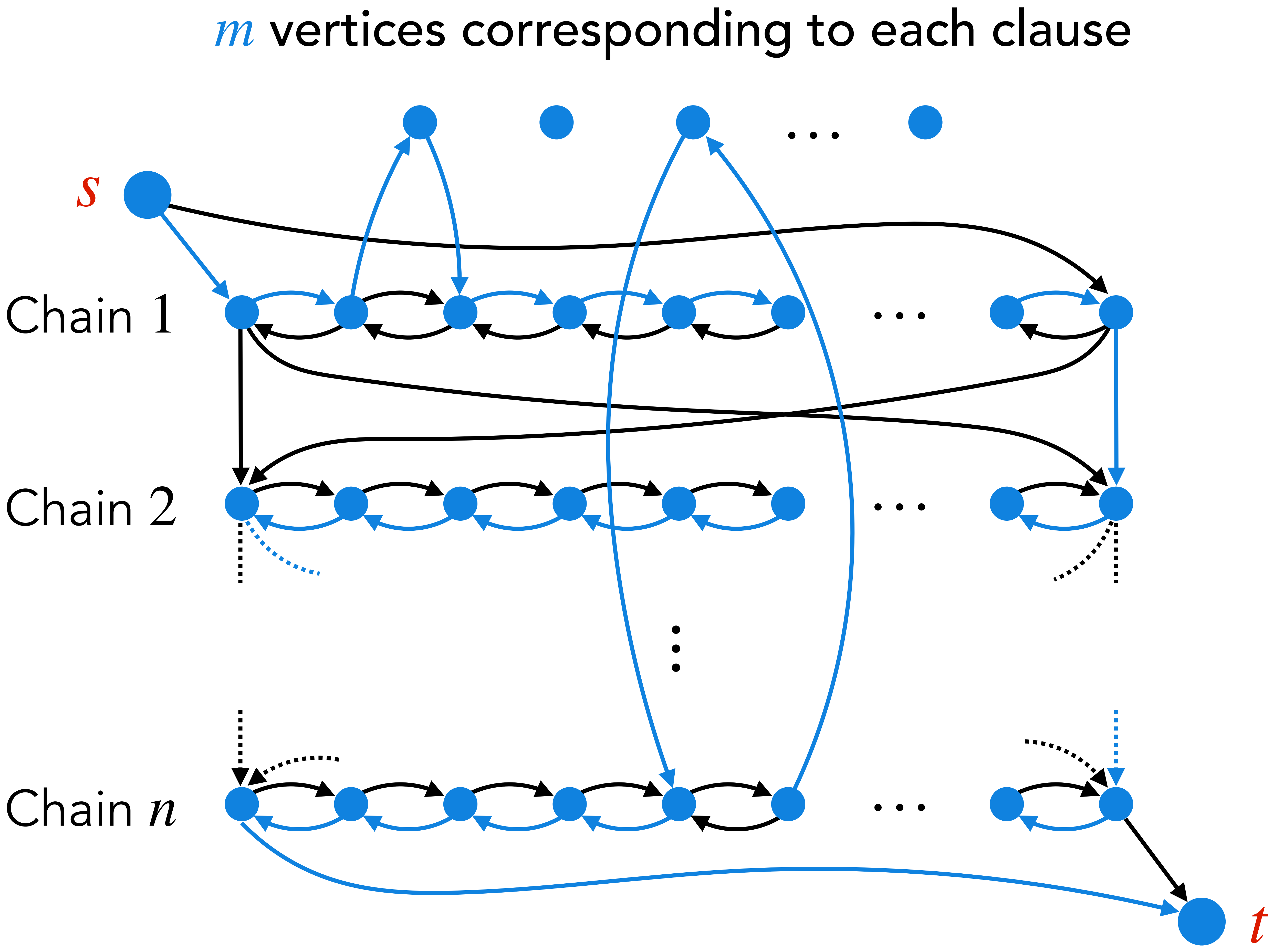
3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

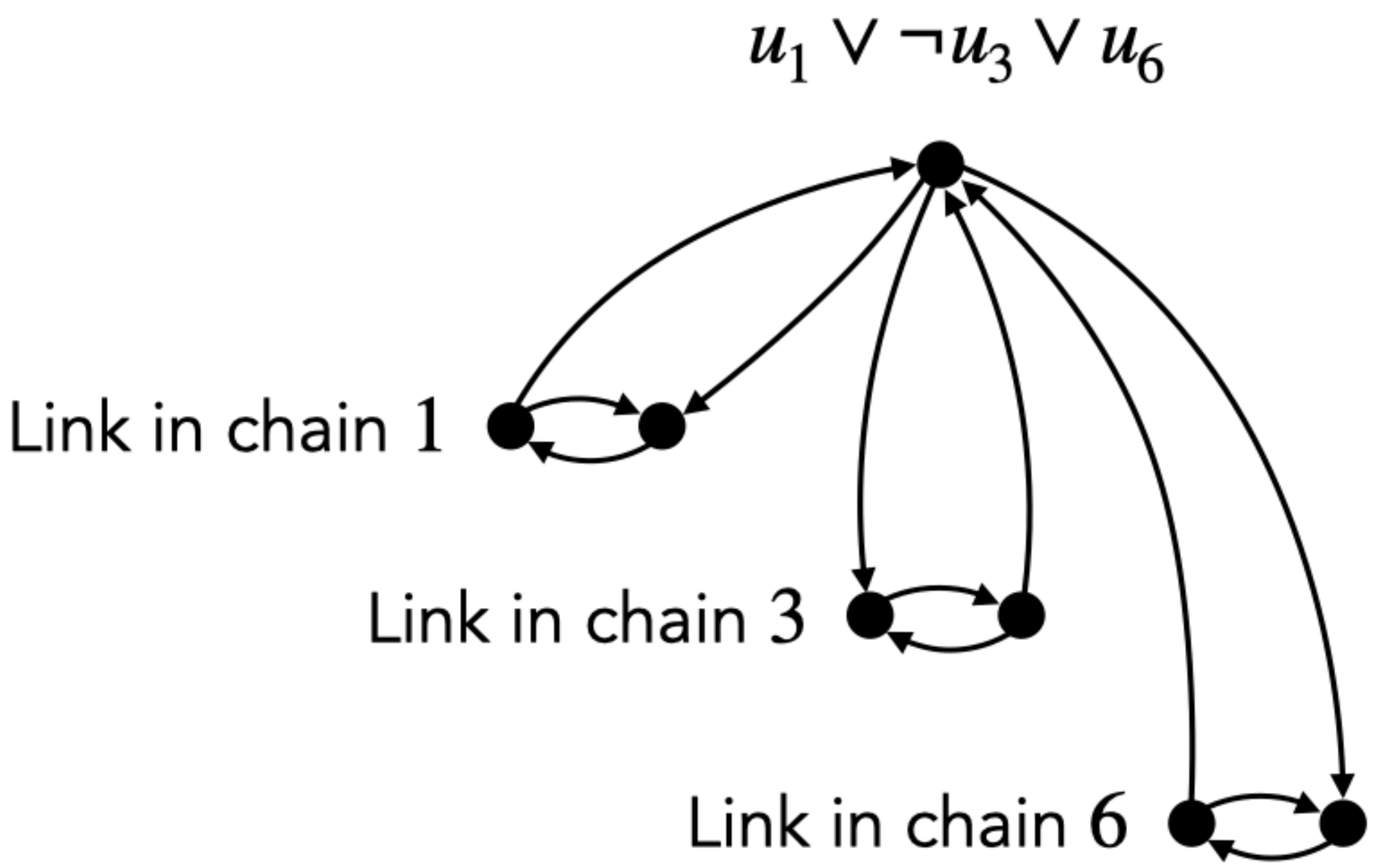
- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
  - $u_i = 1$, if ith chain is traversed left-to-right.
  - $u_i = 0$, if ith chain is traversed right-to-left.
- Why above is a satisfying assignment?

3SAT $\leq_p$ DirHampath

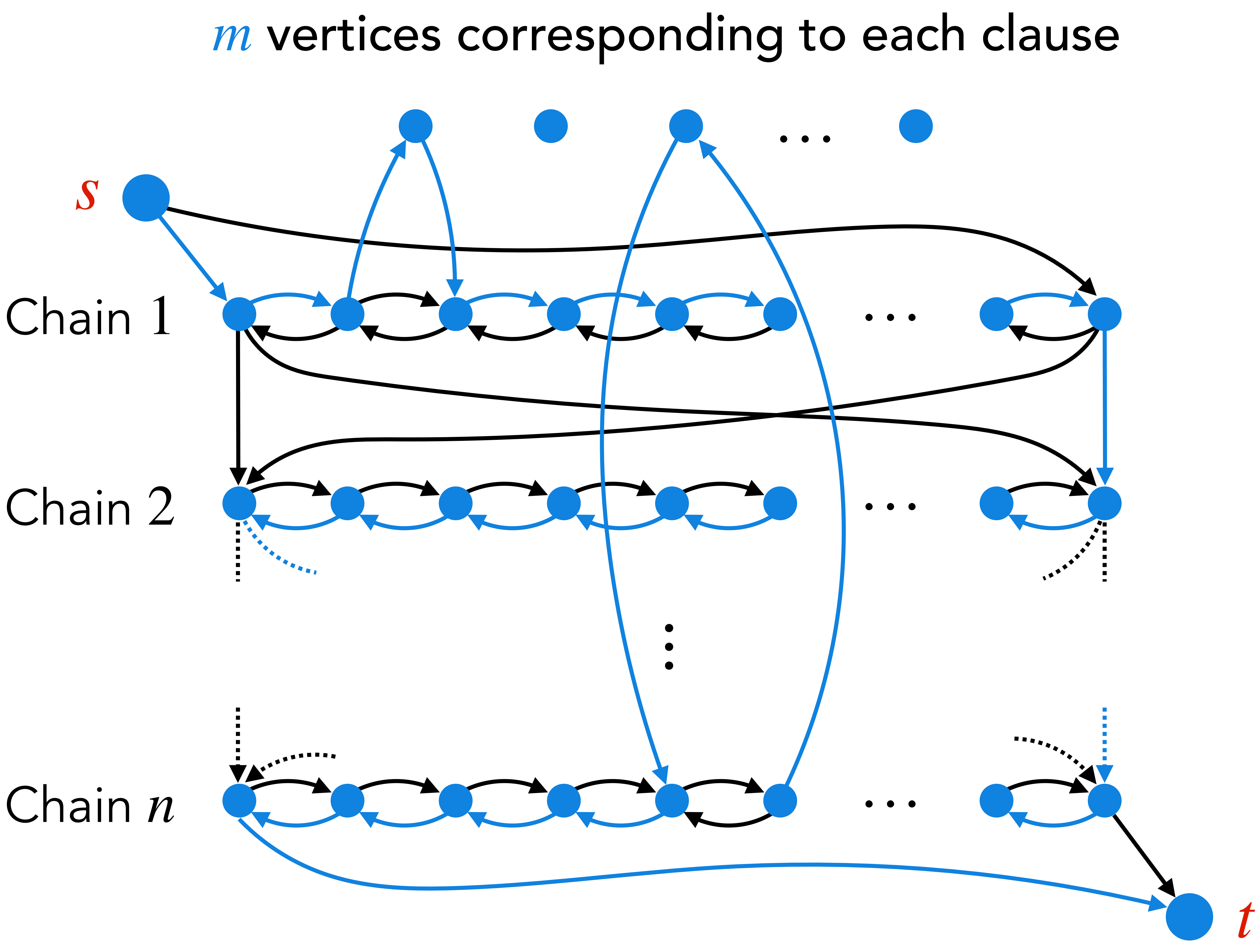


Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
  - $u_i = 1$, if ith chain is traversed left-to-right.
  - $u_i = 0$, if ith chain is traversed right-to-left.
- Why above is a satisfying assignment?

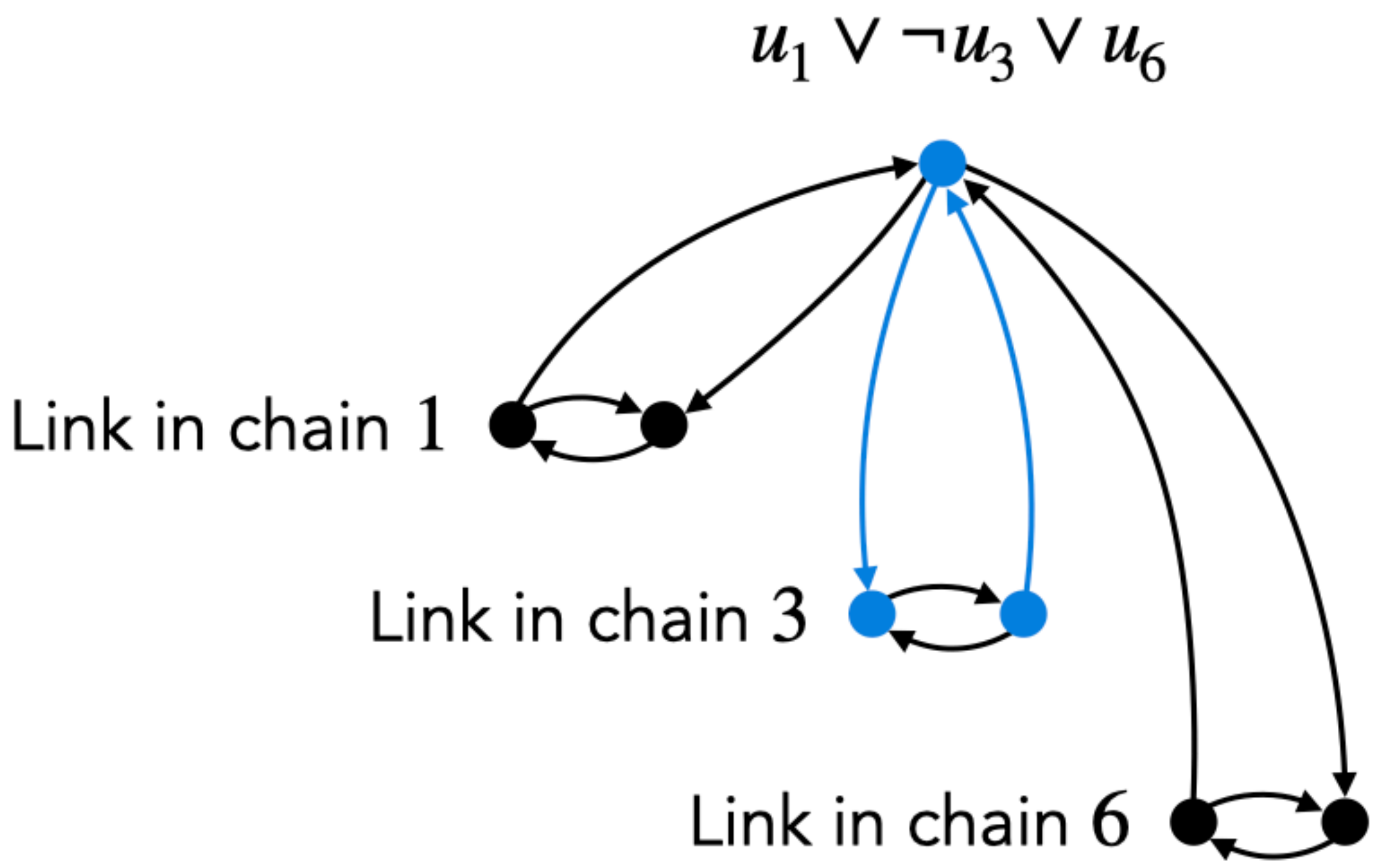


3SAT $\leq_p$ DirHampath

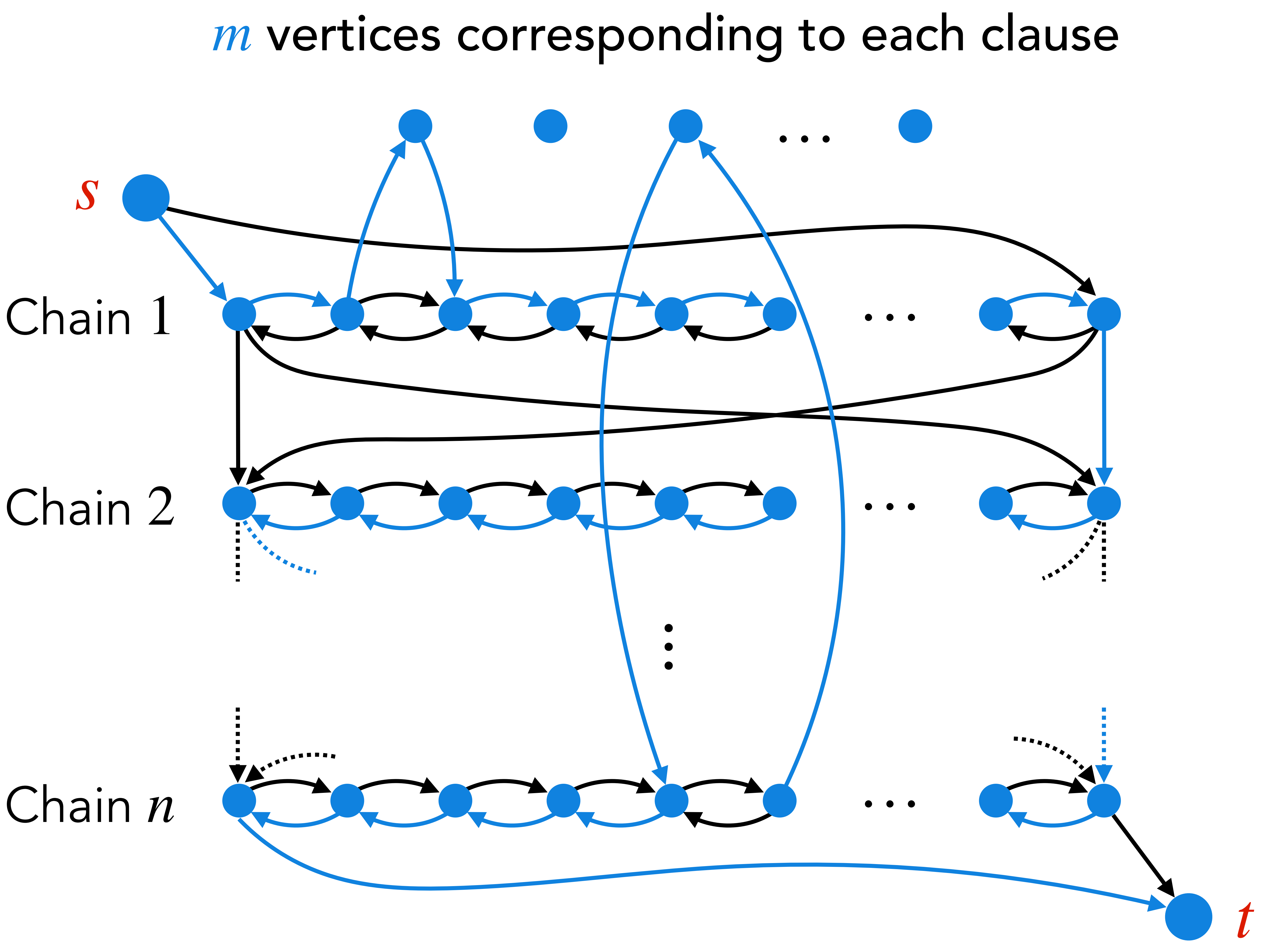


Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
  - $u_i = 1$, if ith chain is traversed left-to-right.
  - $u_i = 0$, if ith chain is traversed right-to-left.
- Why above is a satisfying assignment?

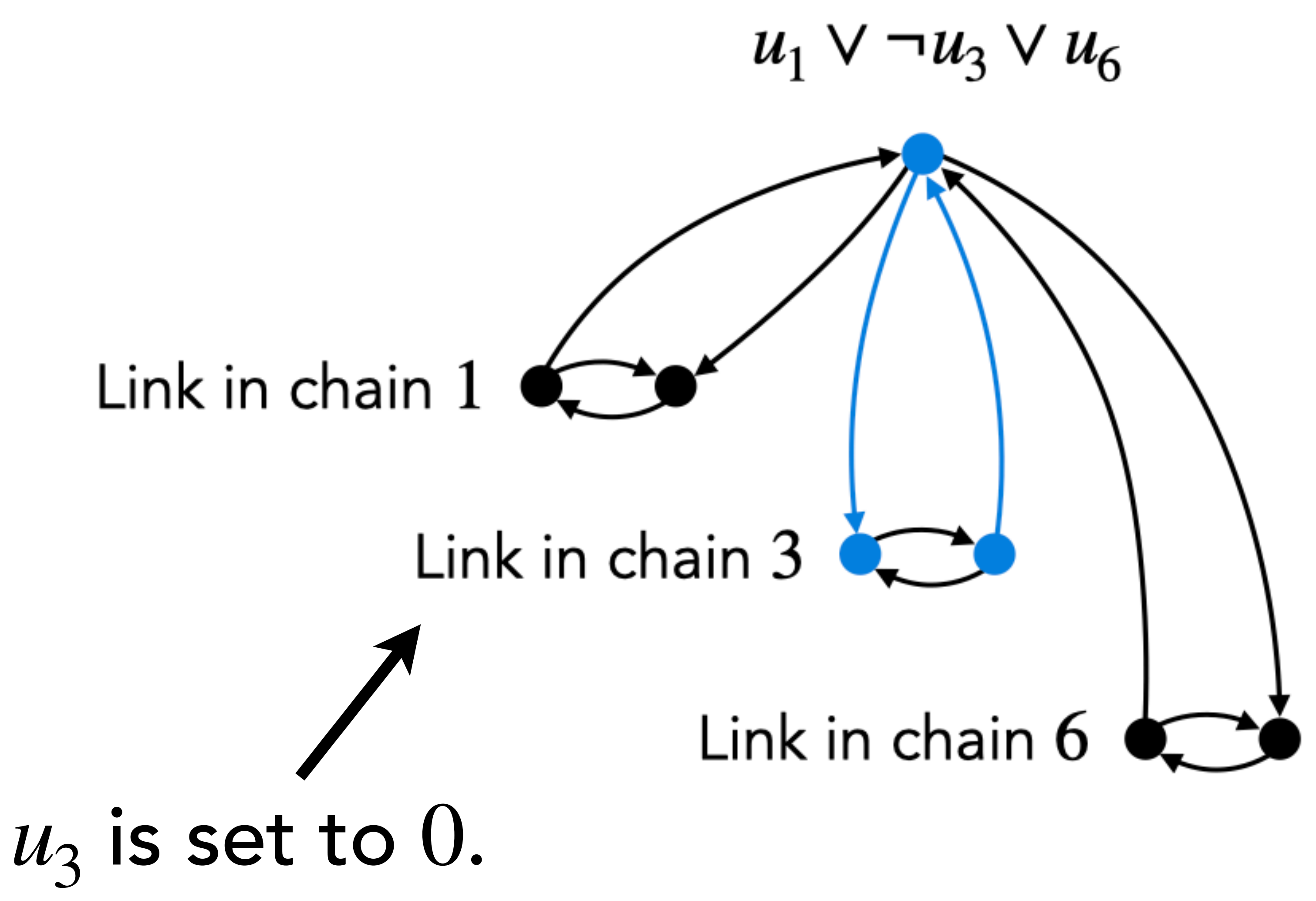


3SAT $\leq_p$ DirHampath



Correctness of Reduction ($\Longleftarrow$):

- Suppose there exists a hamiltonian st-path.
- Satisfying assignment for ϕ:
  - $u_i = 1$, if ith chain is traversed left-to-right.
  - $u_i = 0$, if ith chain is traversed right-to-left.
- Why above is a satisfying assignment?



DirHampath $\leq_p$ *Hampath*

DirHampath $\leq_p$ *Hampath*

- *DirHampath* $= \{\langle G, s, t\rangle \mid G$ is a directed graph with a hamiltonian path from s to $t\}$
- *Hampath* $= \{\langle G', s', t'\rangle \mid G'$ is an undirected graph with a hamiltonian path from s' to $t'\}$

DirHampath $\leq_p$ *Hampath*

- *DirHampath* $= \{\langle G, s, t\rangle \mid G$ is a directed graph with a hamiltonian path from s to $t\}$
- *Hampath* $= \{\langle G', s', t'\rangle \mid G'$ is an undirected graph with a hamiltonian path from s' to $t'\}$

Goal: Convert $\langle G, s, t\rangle$ into $\langle G', s', t'\rangle$ in polytime, such that G has a hamiltonian path from s to t

DirHampath $\leq_p$ *Hampath*

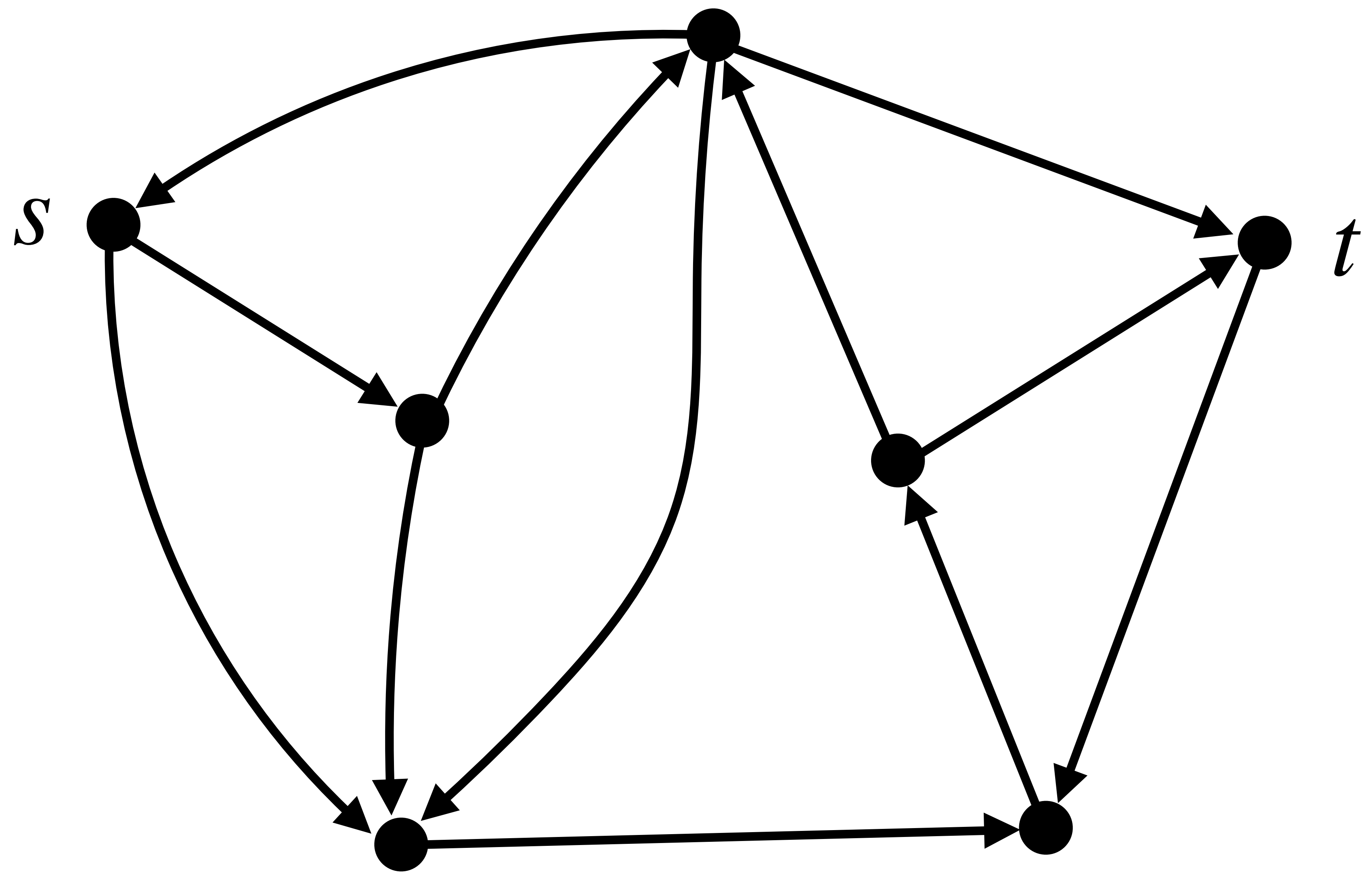
- *DirHampath* $= \{\langle G, s, t\rangle \mid G$ is a directed graph with a hamiltonian path from s to $t\}$
- *Hampath* $= \{\langle G', s', t'\rangle \mid G'$ is an undirected graph with a hamiltonian path from s' to $t'\}$

Goal: Convert $\langle G, s, t\rangle$ into $\langle G', s', t'\rangle$ in polytime, such that G has a hamiltonian path from s to t if and only if G' has a hamiltonian path from s' to t'.

DirHampath $\leq_p$ *Hampath*

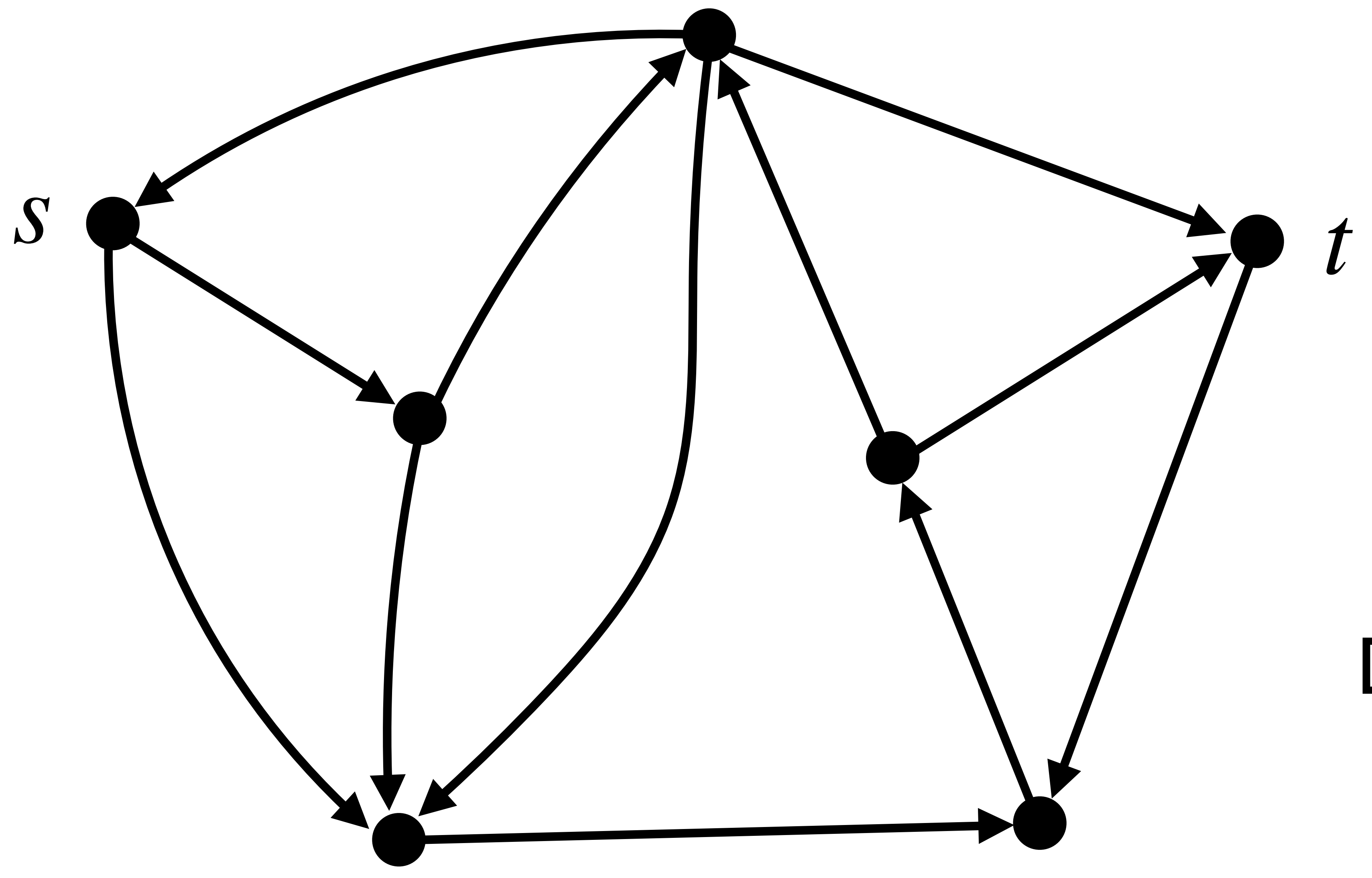
DirHampath $\leq_p$ *Hampath*

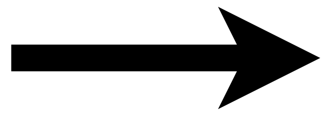
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$**:**

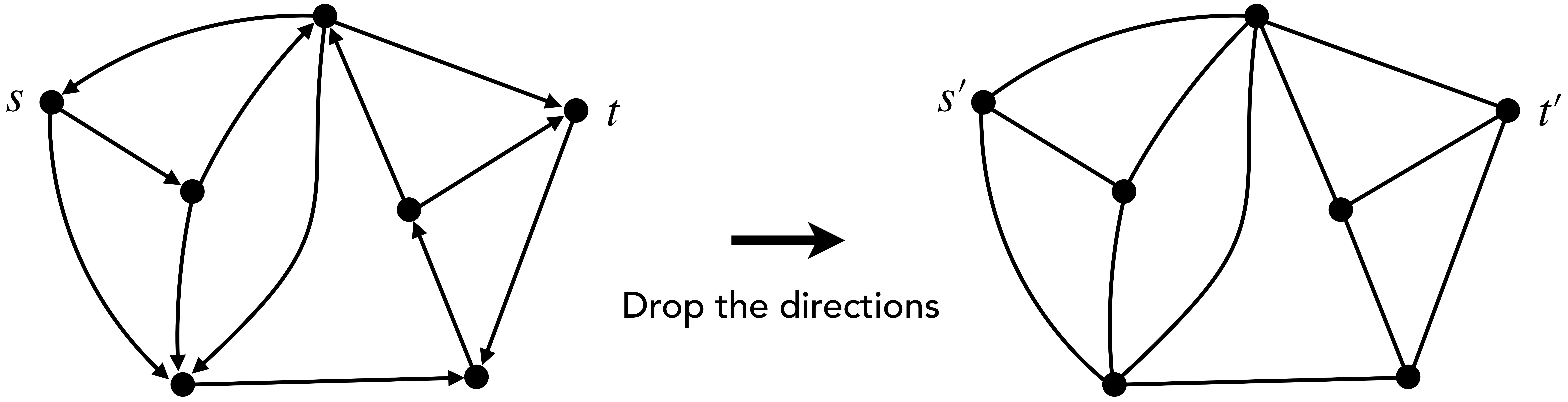




Drop the directions

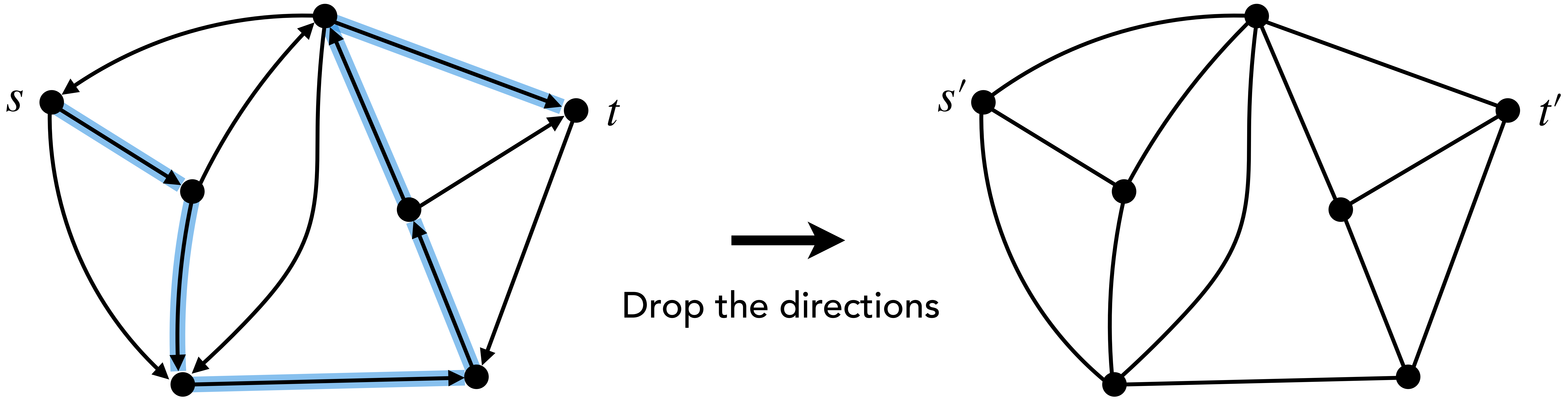
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



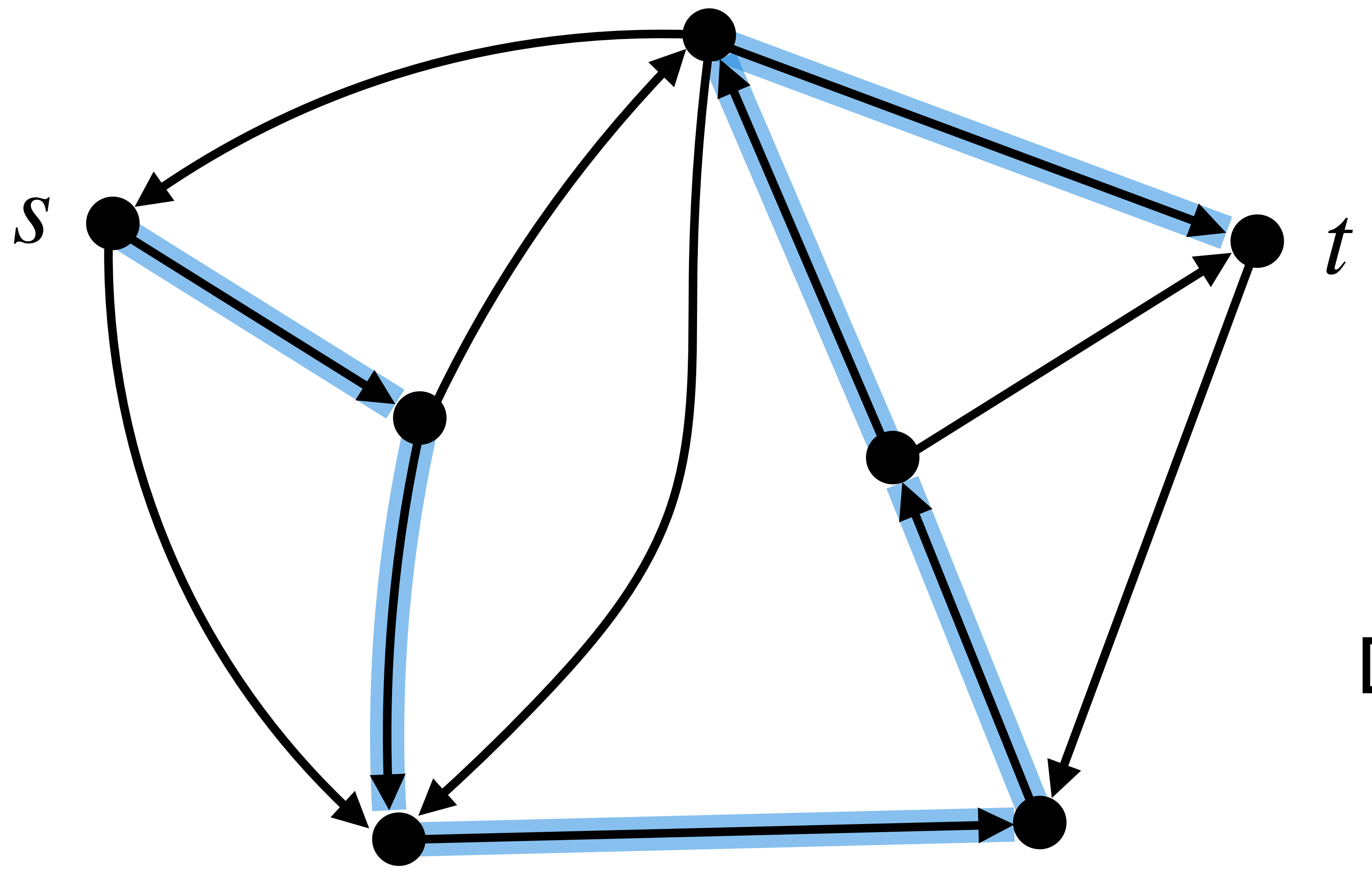
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:

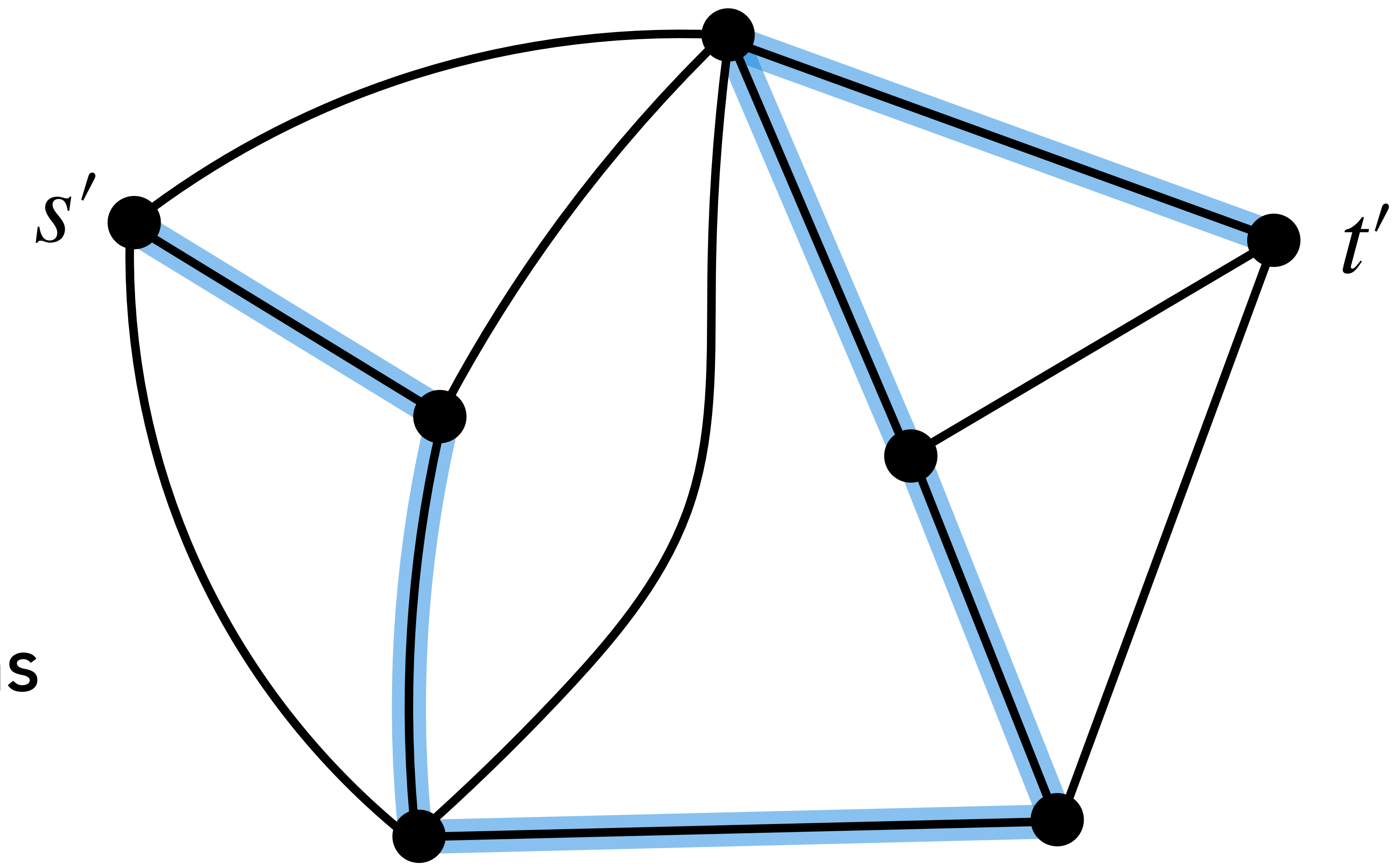


DirHampath $\leq_p$ *Hampath*

$\langle G, s, t\rangle \rightarrow \langle G', s', t'\rangle$:

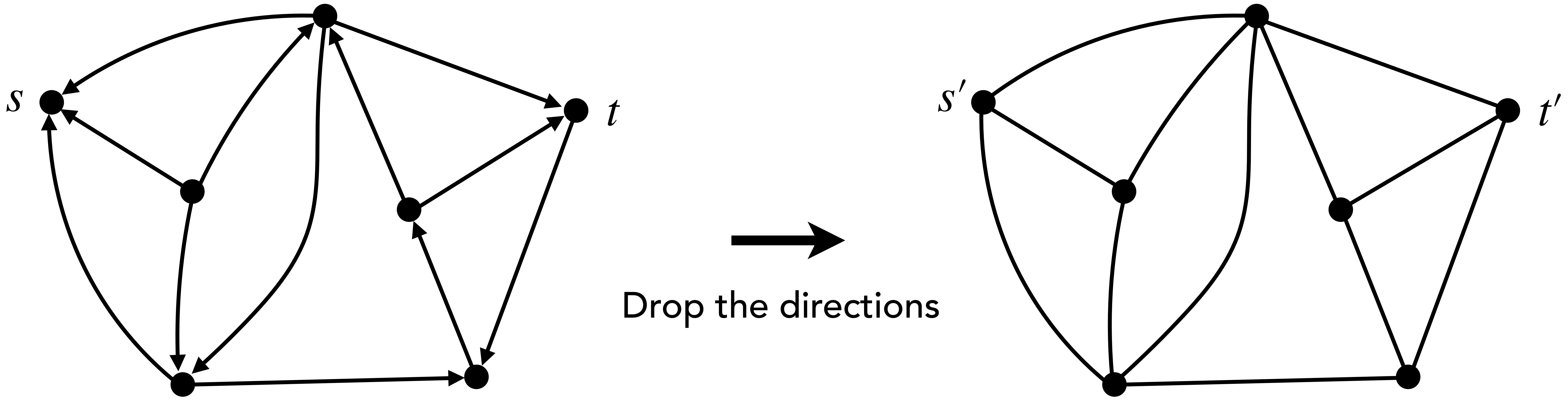


Drop the directions



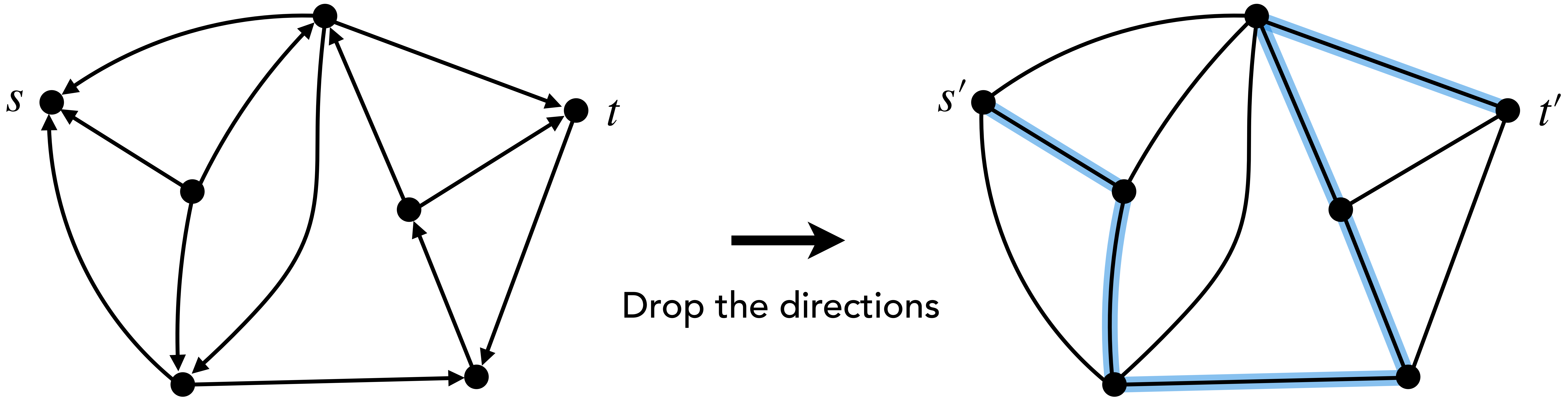
$DirHampath \leq_p Hampath$

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



$DirHampath \leq_p Hampath$

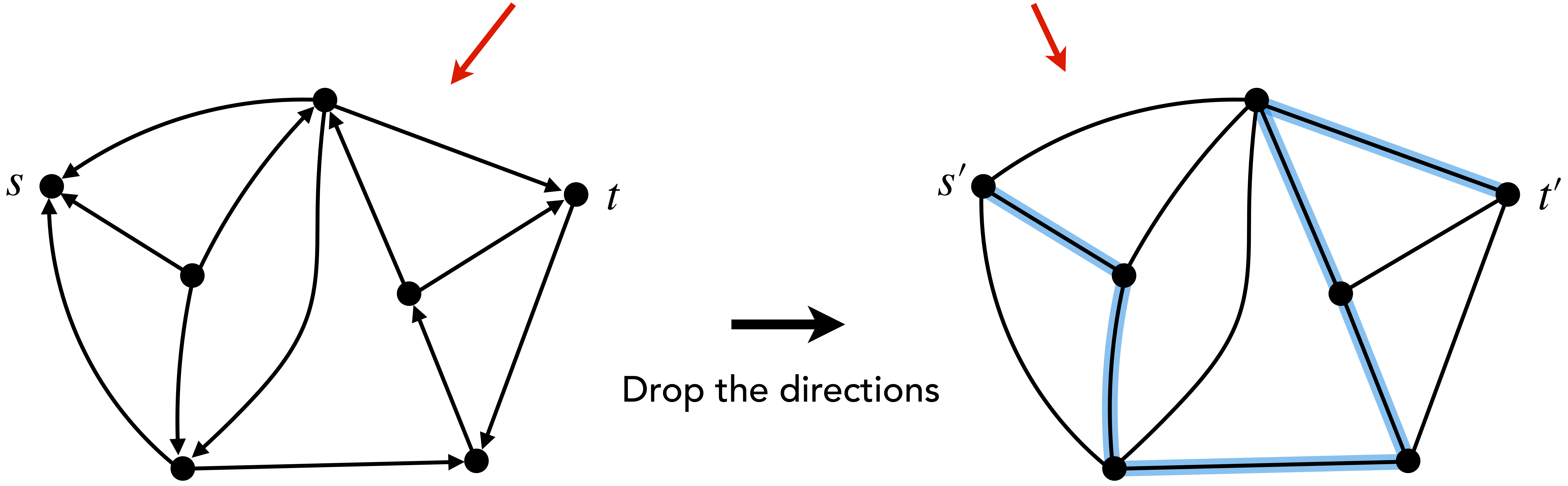
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:

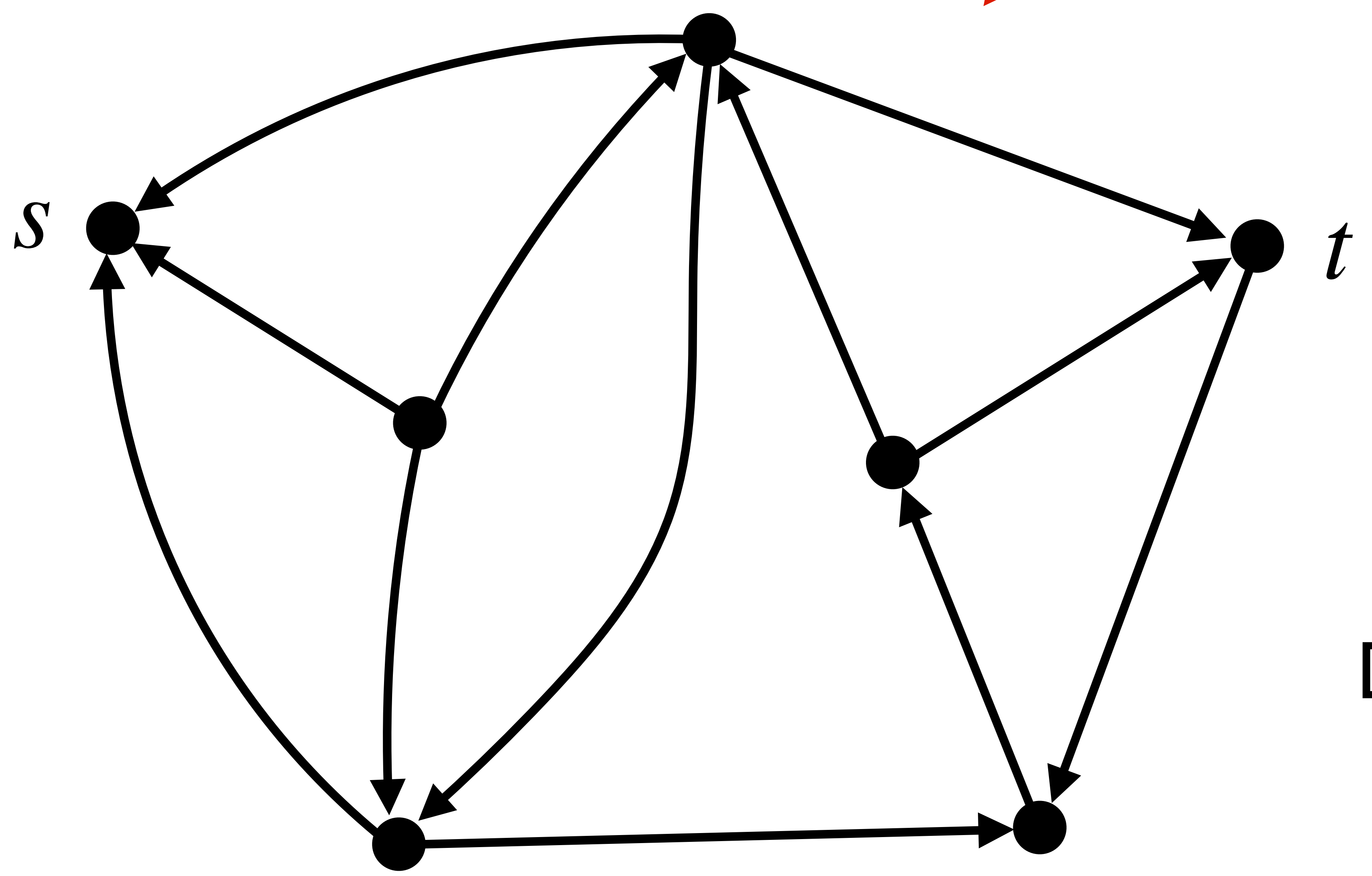
No hamiltonian st-path in G, but a hamiltonian $s't'$-path in G'.



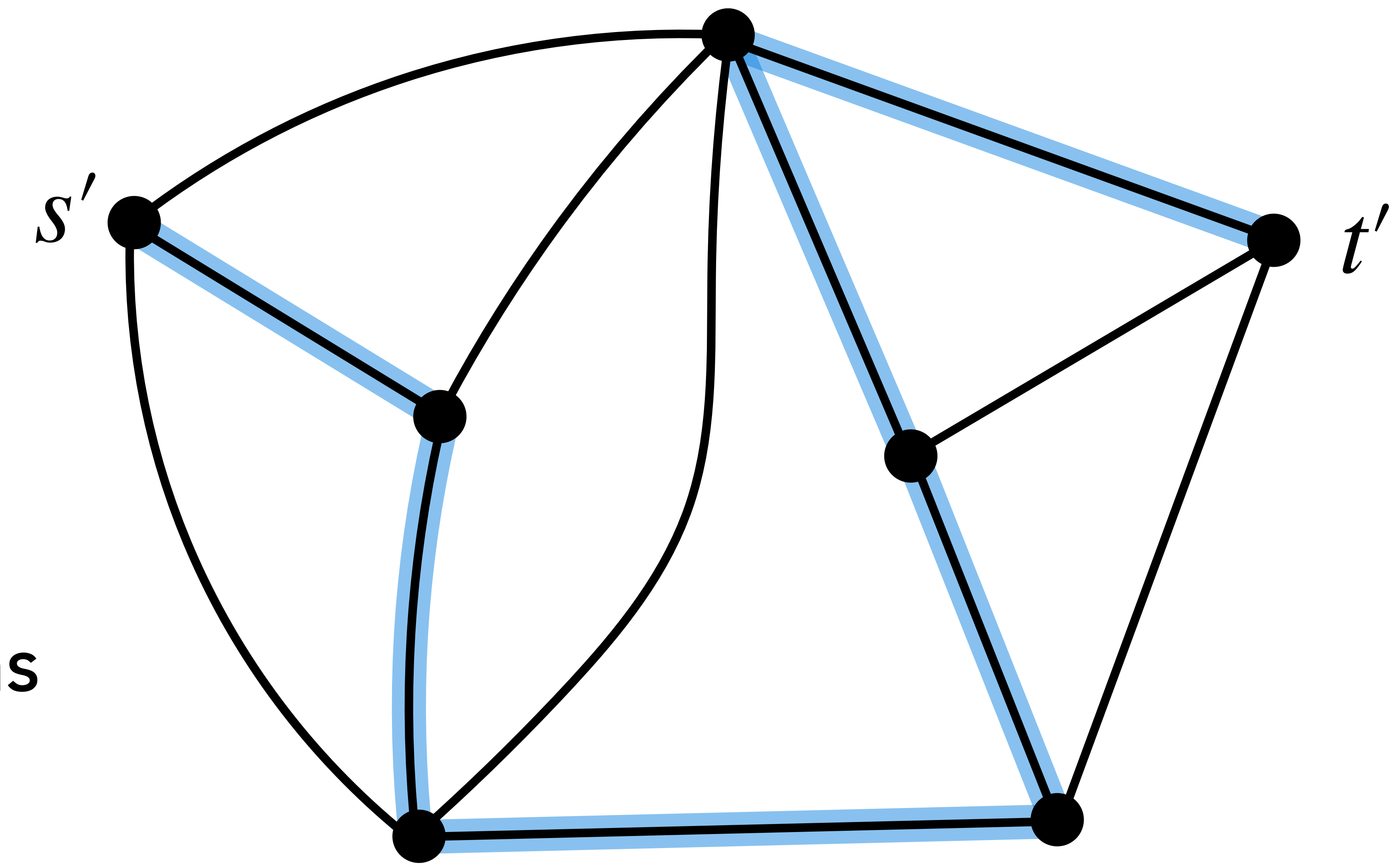
$DirHampath \leq_p Hampath$

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:

Reduction should enable us to find a similar hamiltonian st-path in G w.r.t a hamiltonian $s't'$-path in G'.



Drop the directions

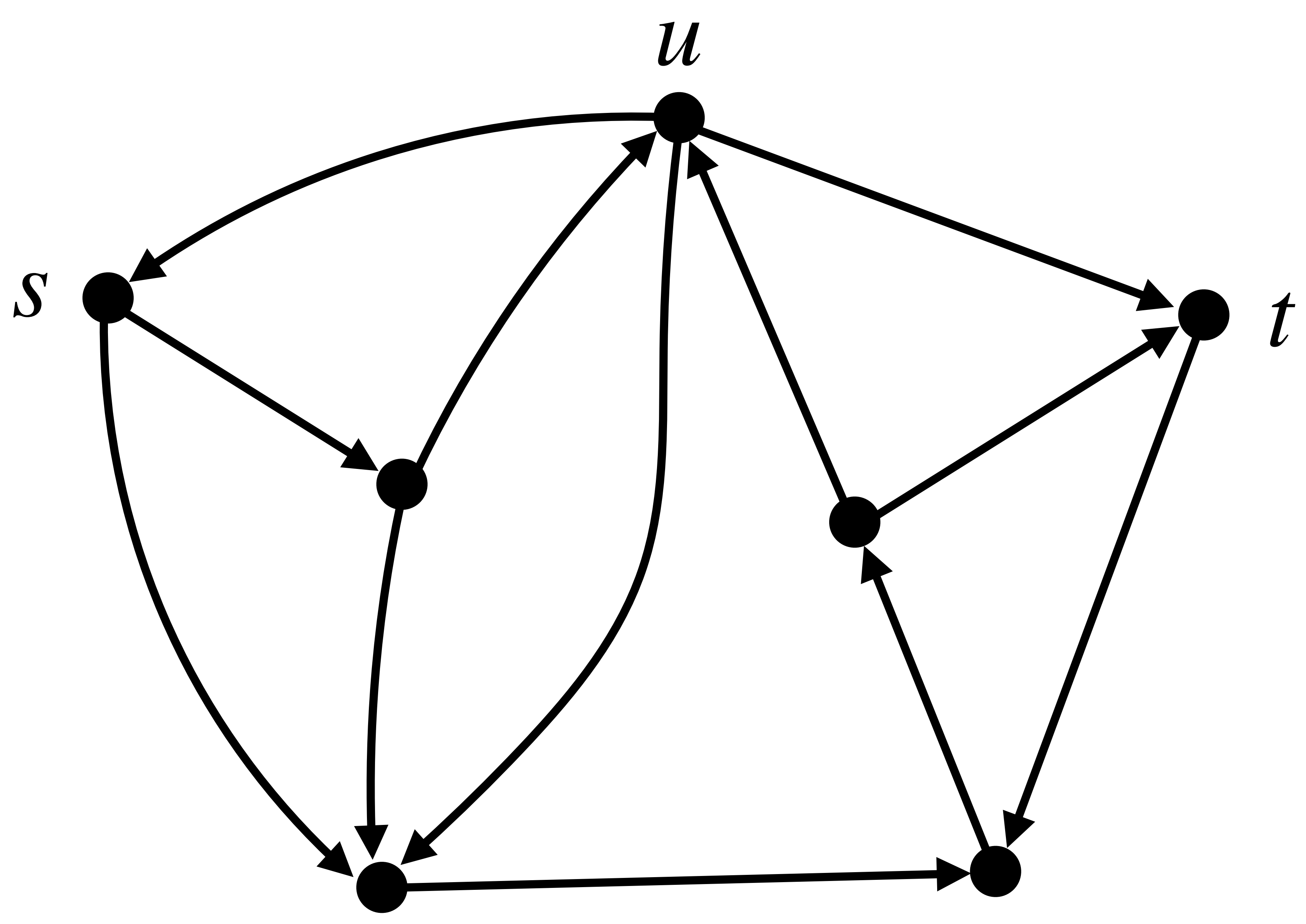


$DirHampath \leq_p Hampath$

$\langle G, s, t\rangle \rightarrow \langle G', s', t'\rangle$:

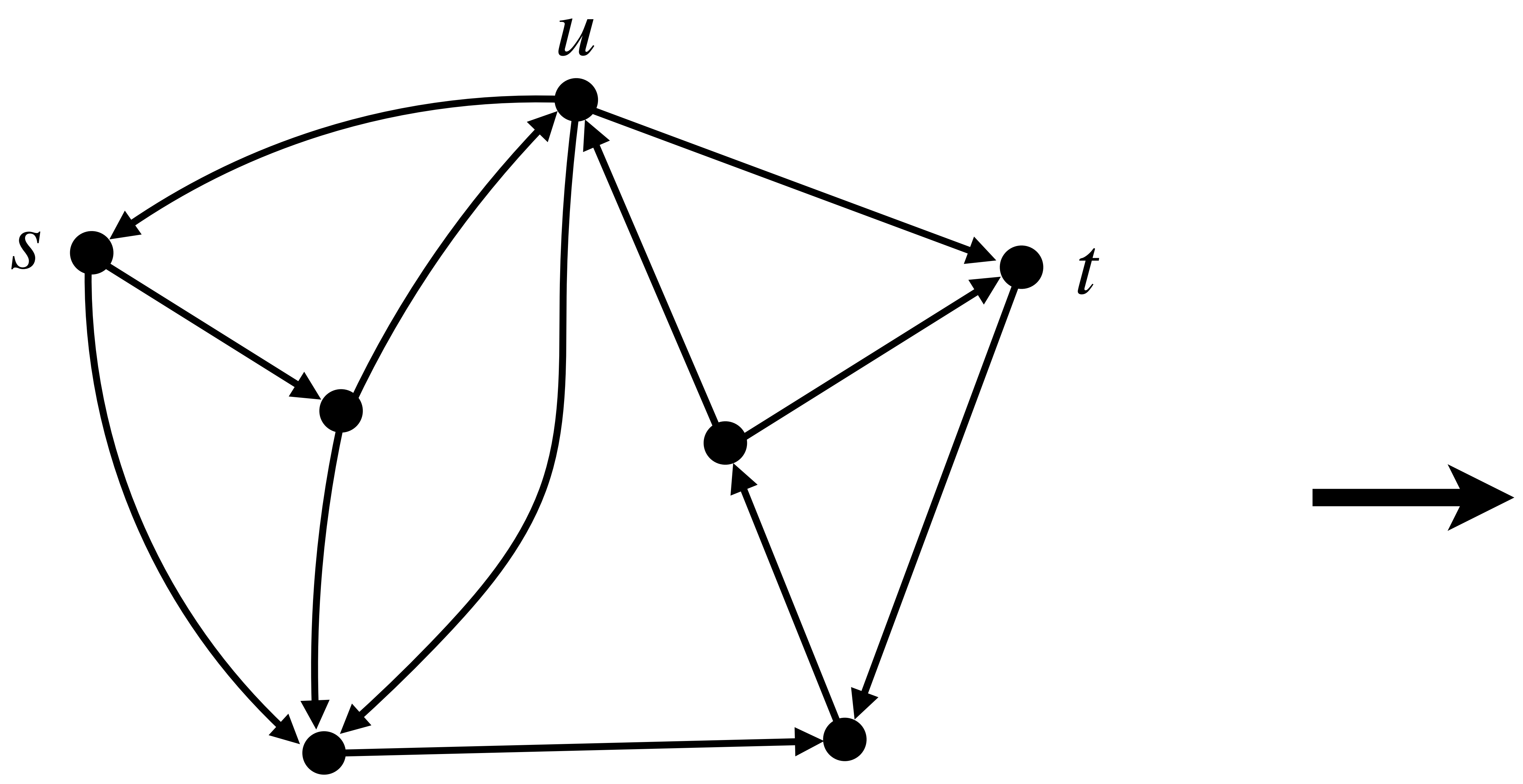
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



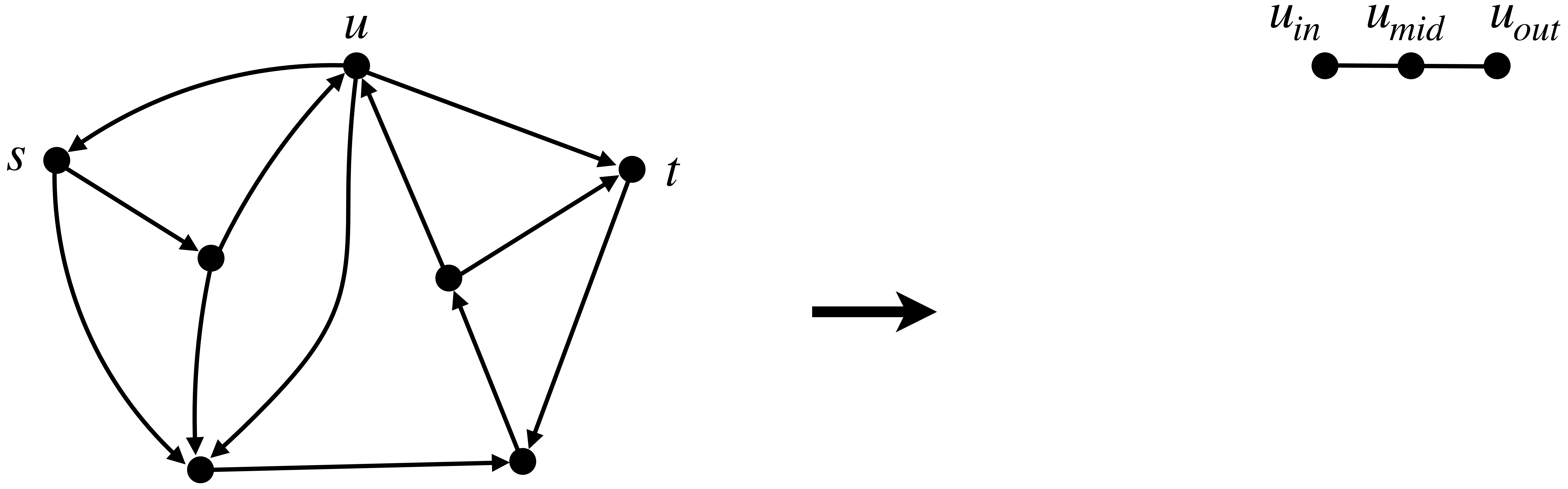
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$**:**



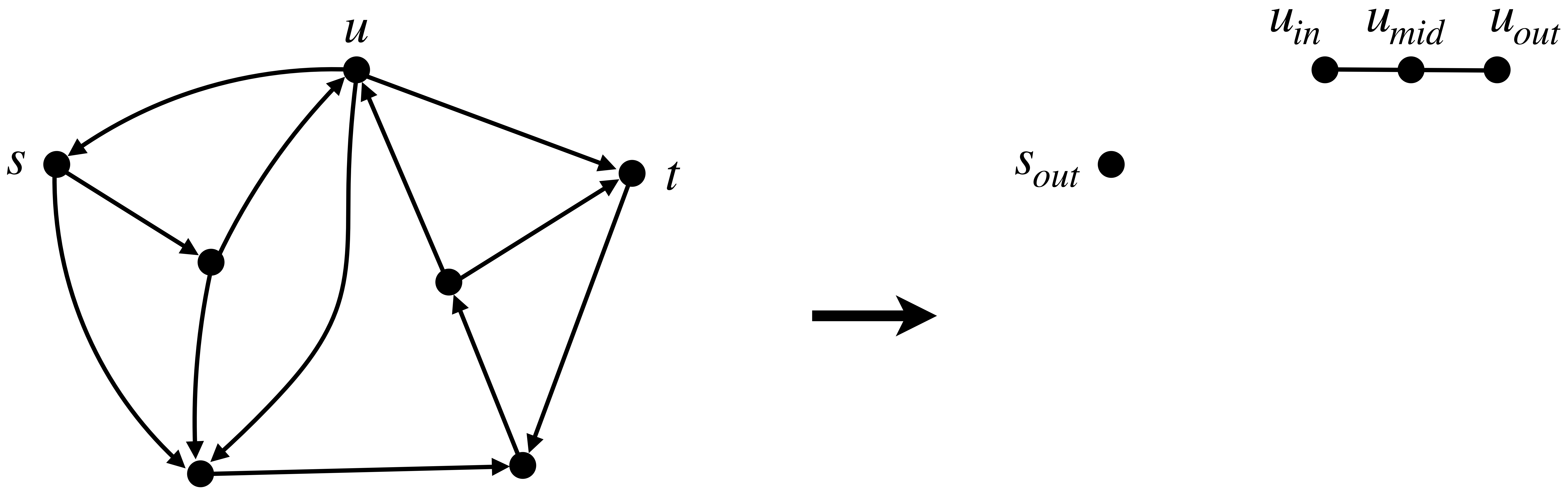
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$**:**



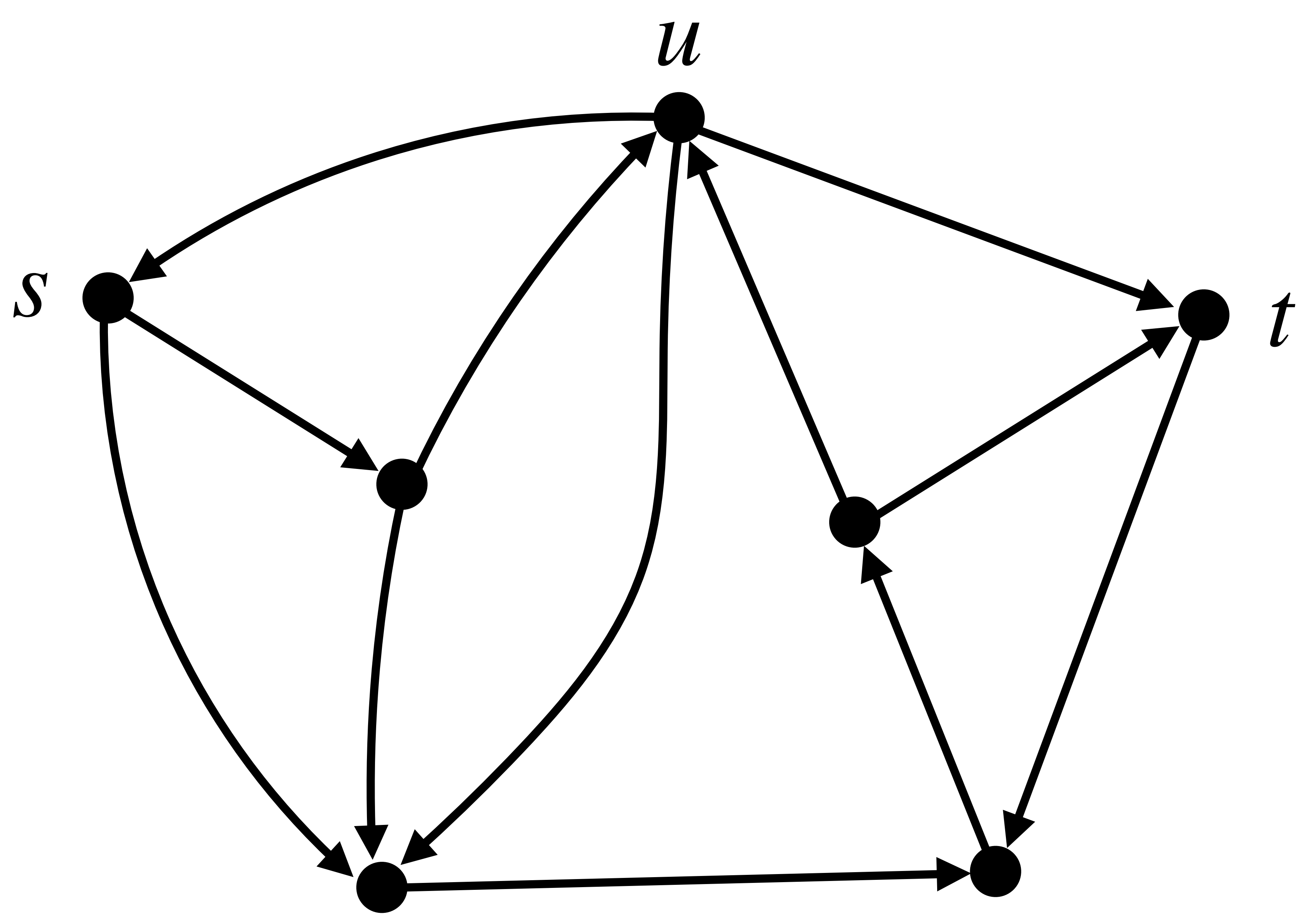
DirHampath $\leq_p$ *Hampath*

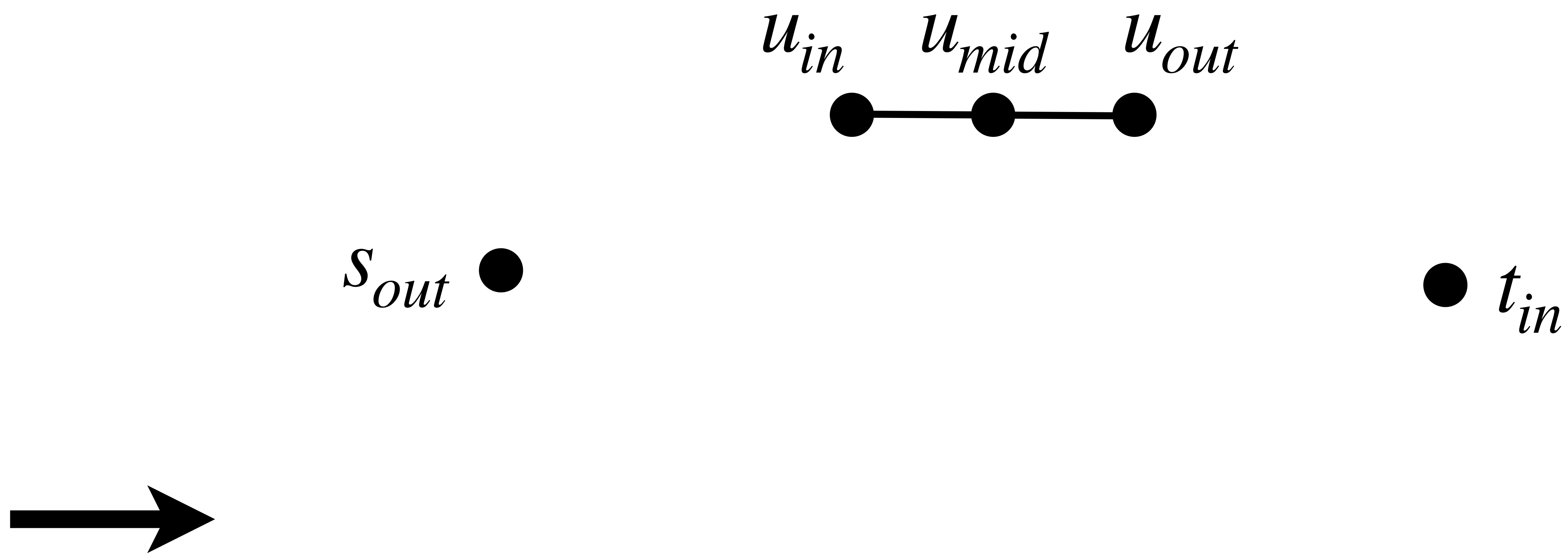
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



DirHampath $\leq_p$ Hampath

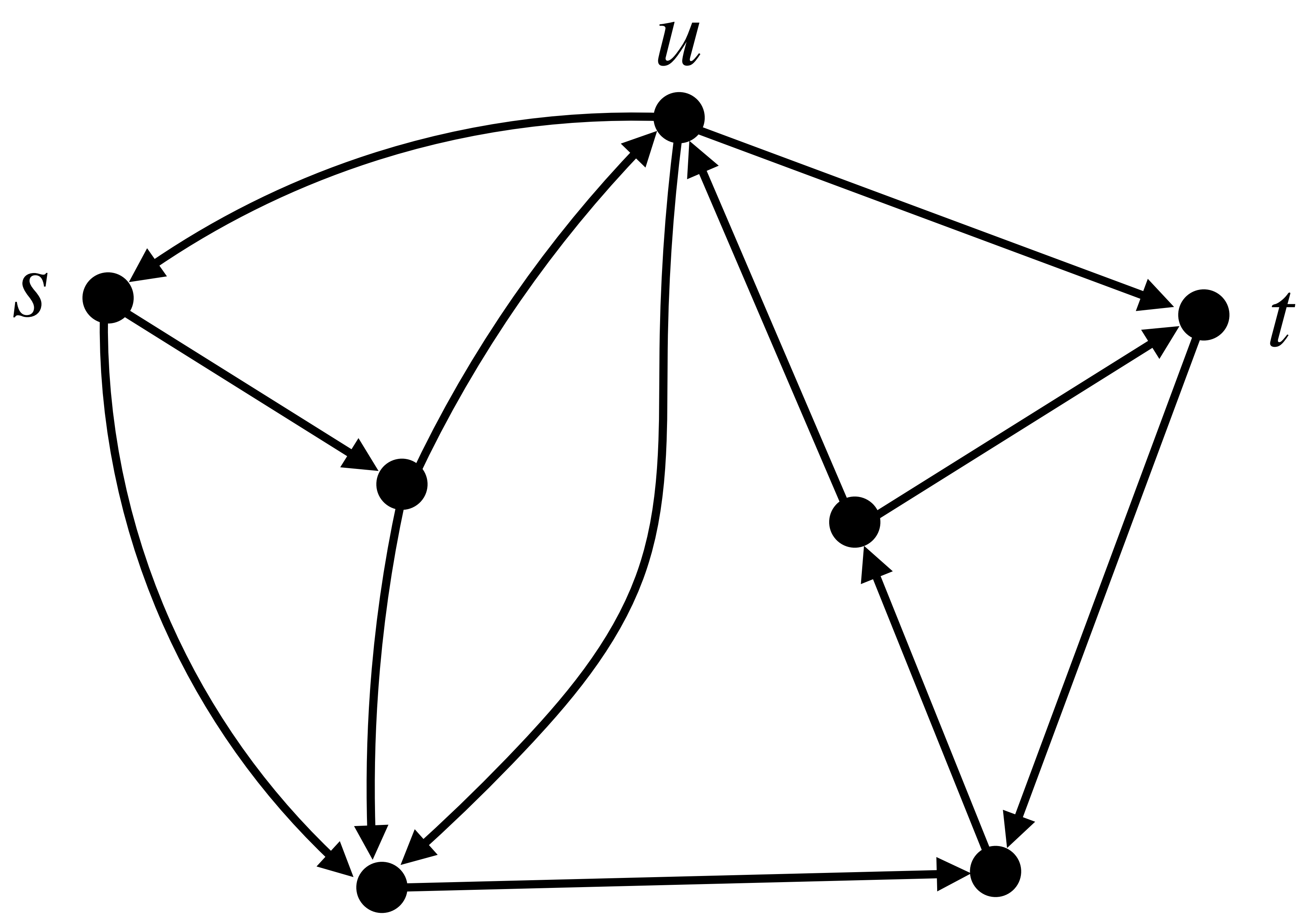
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:

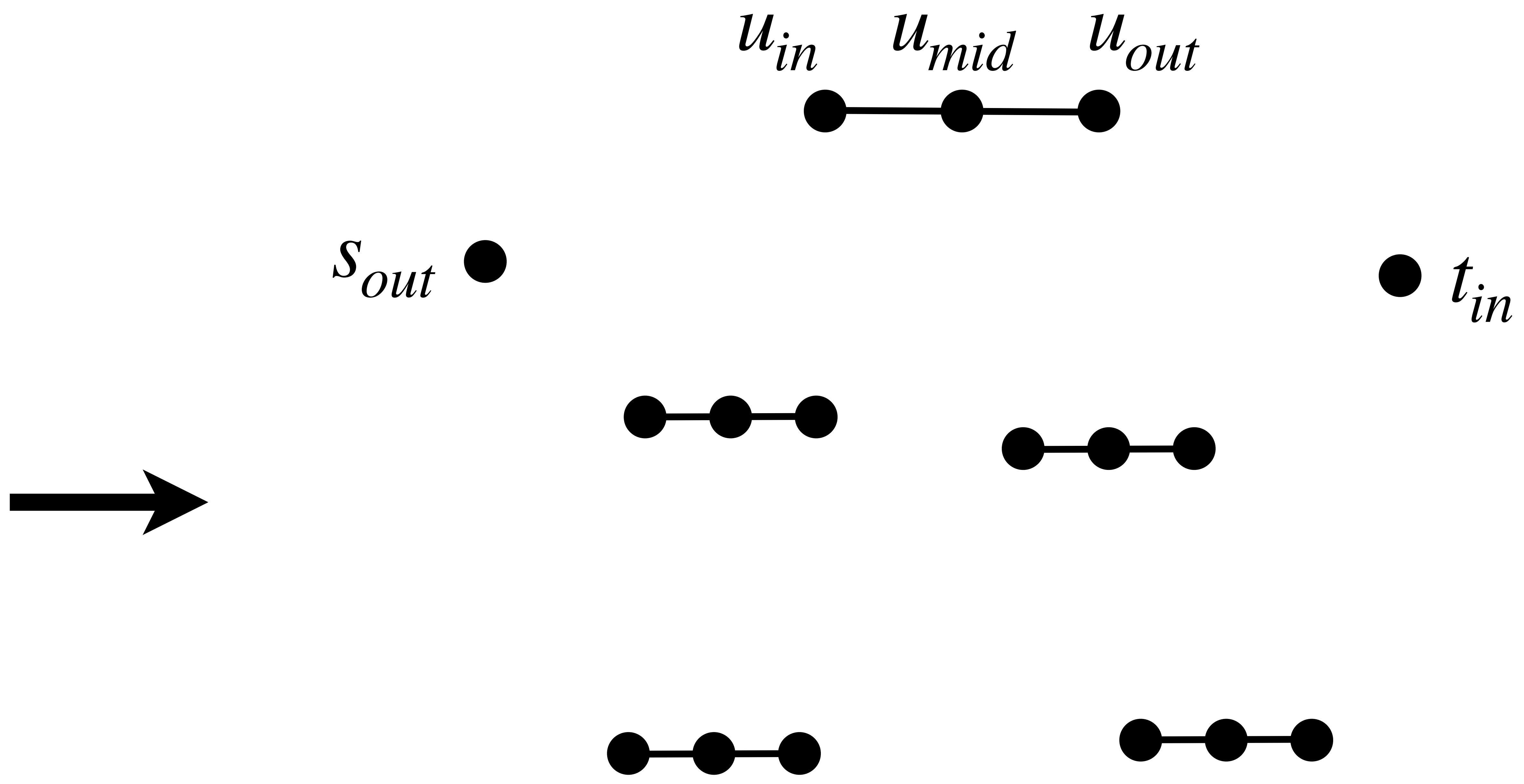




DirHampath $\leq_p$ *Hampath*

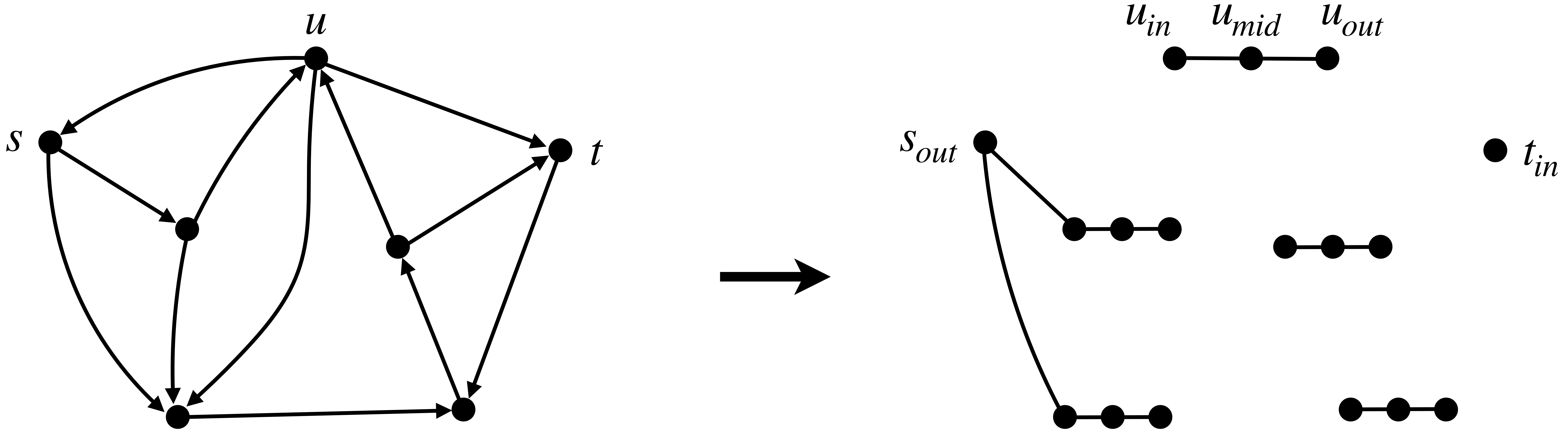
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:





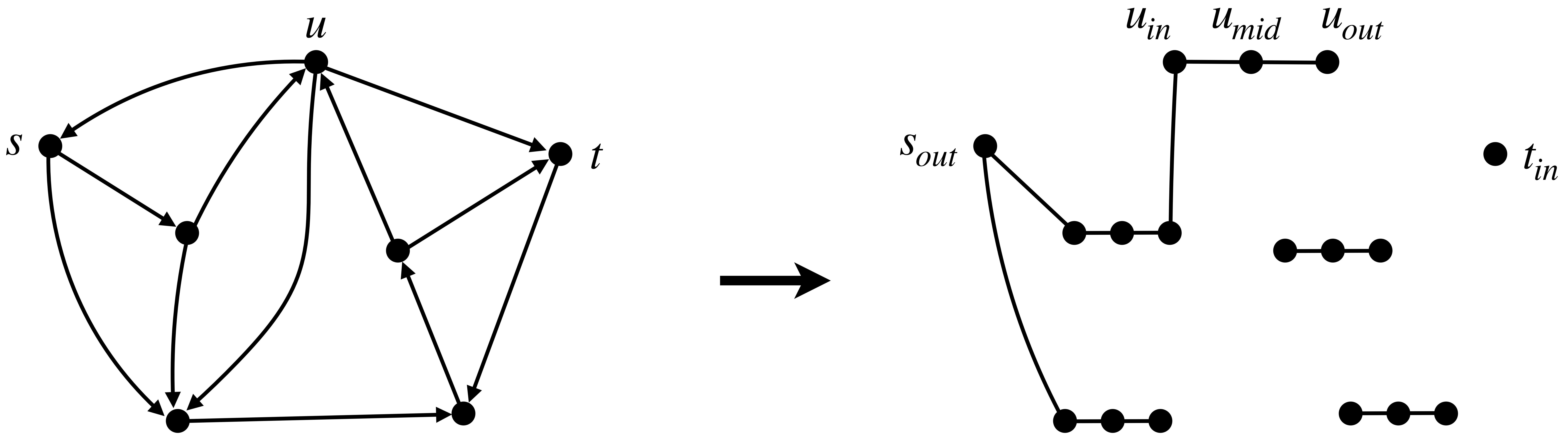
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:



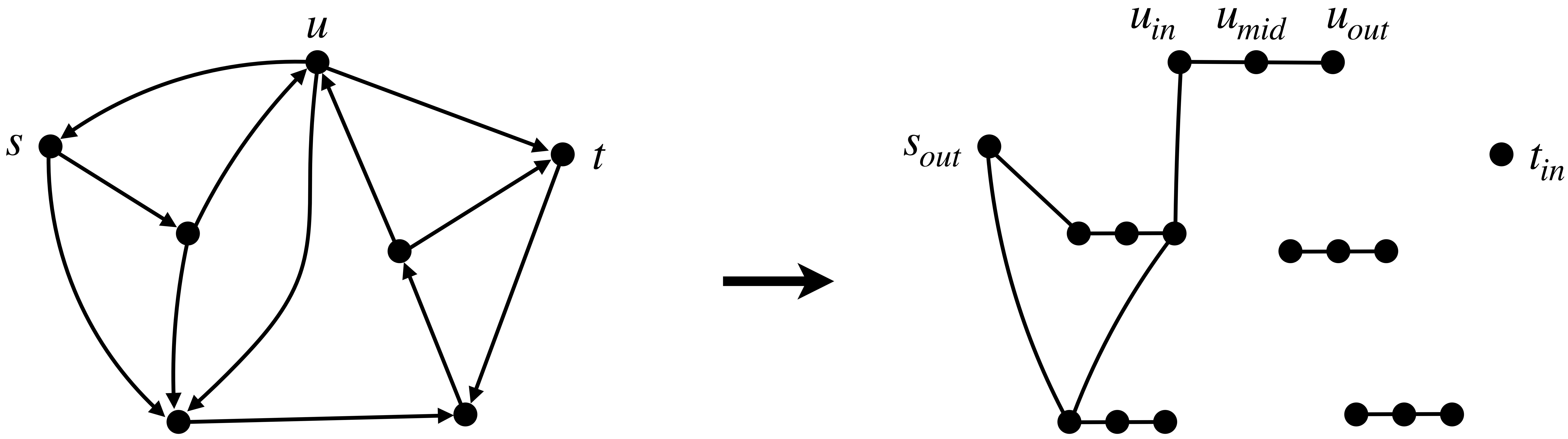
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



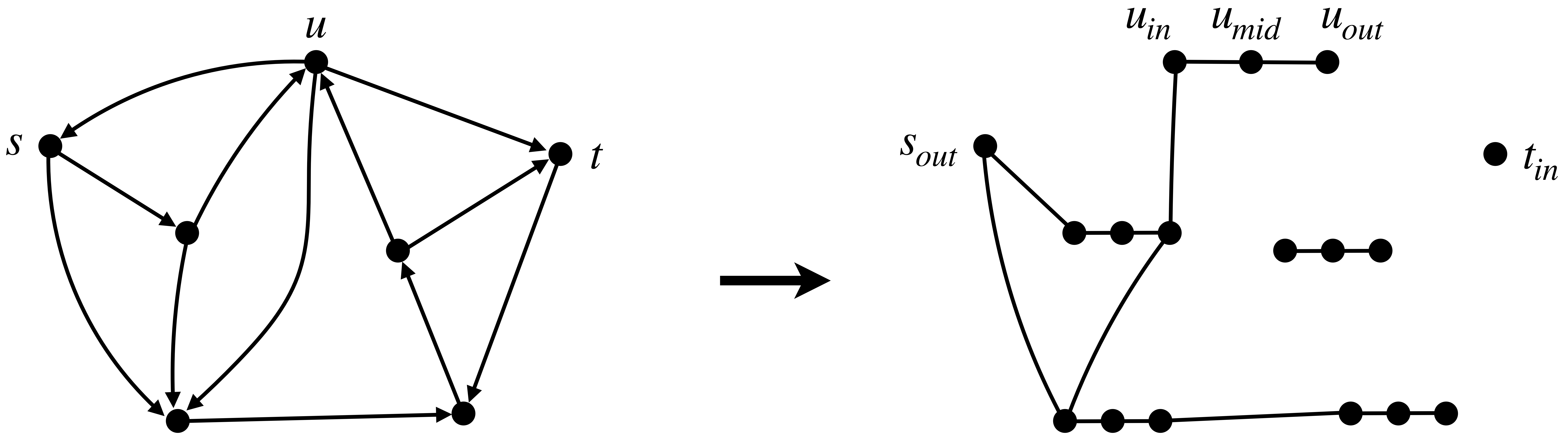
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$**:**



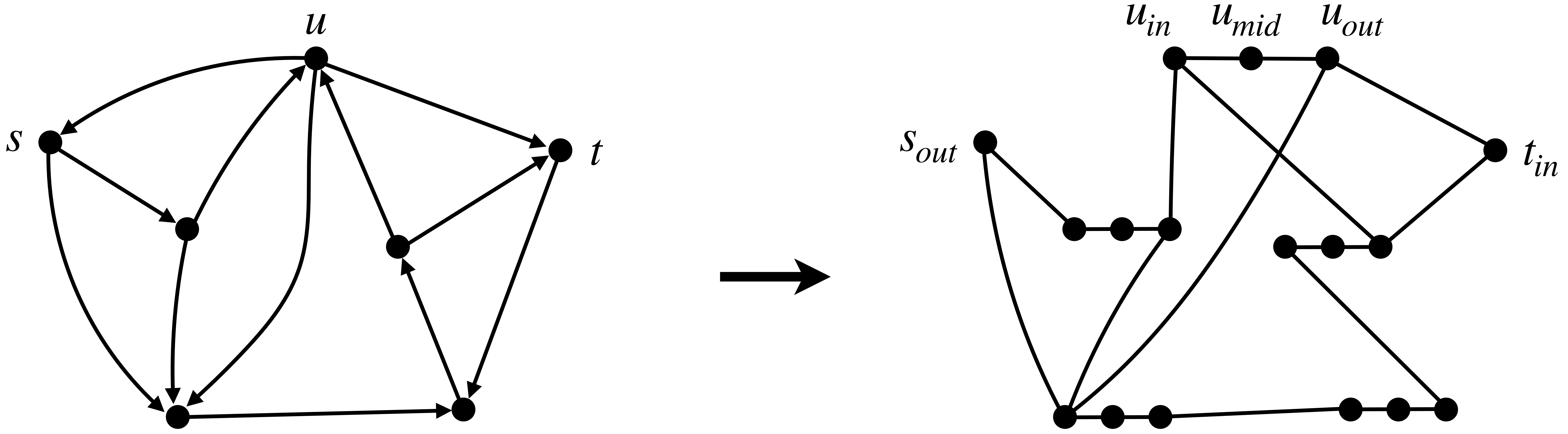
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:



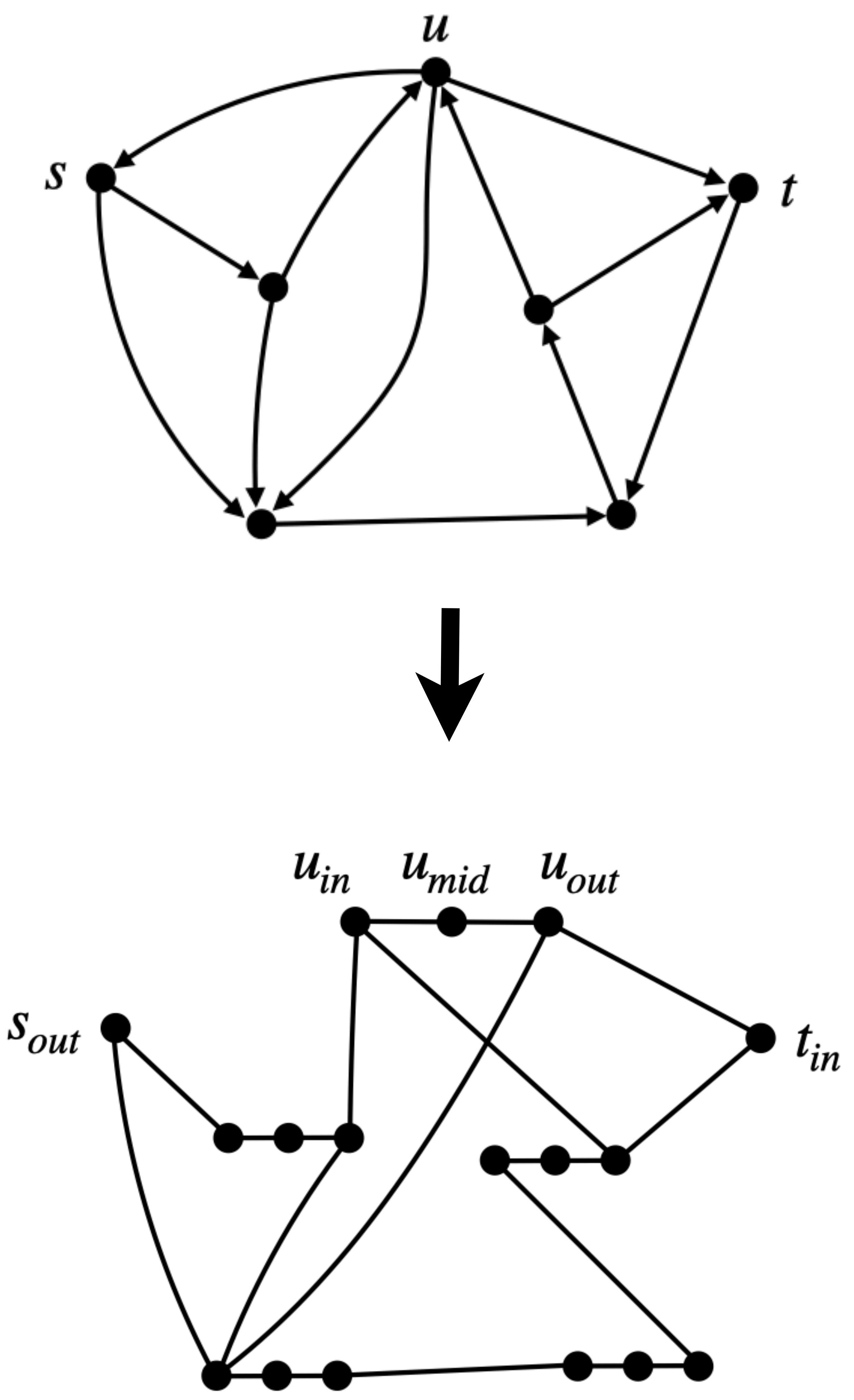
DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:



$DirHampath \leq_p Hampath$

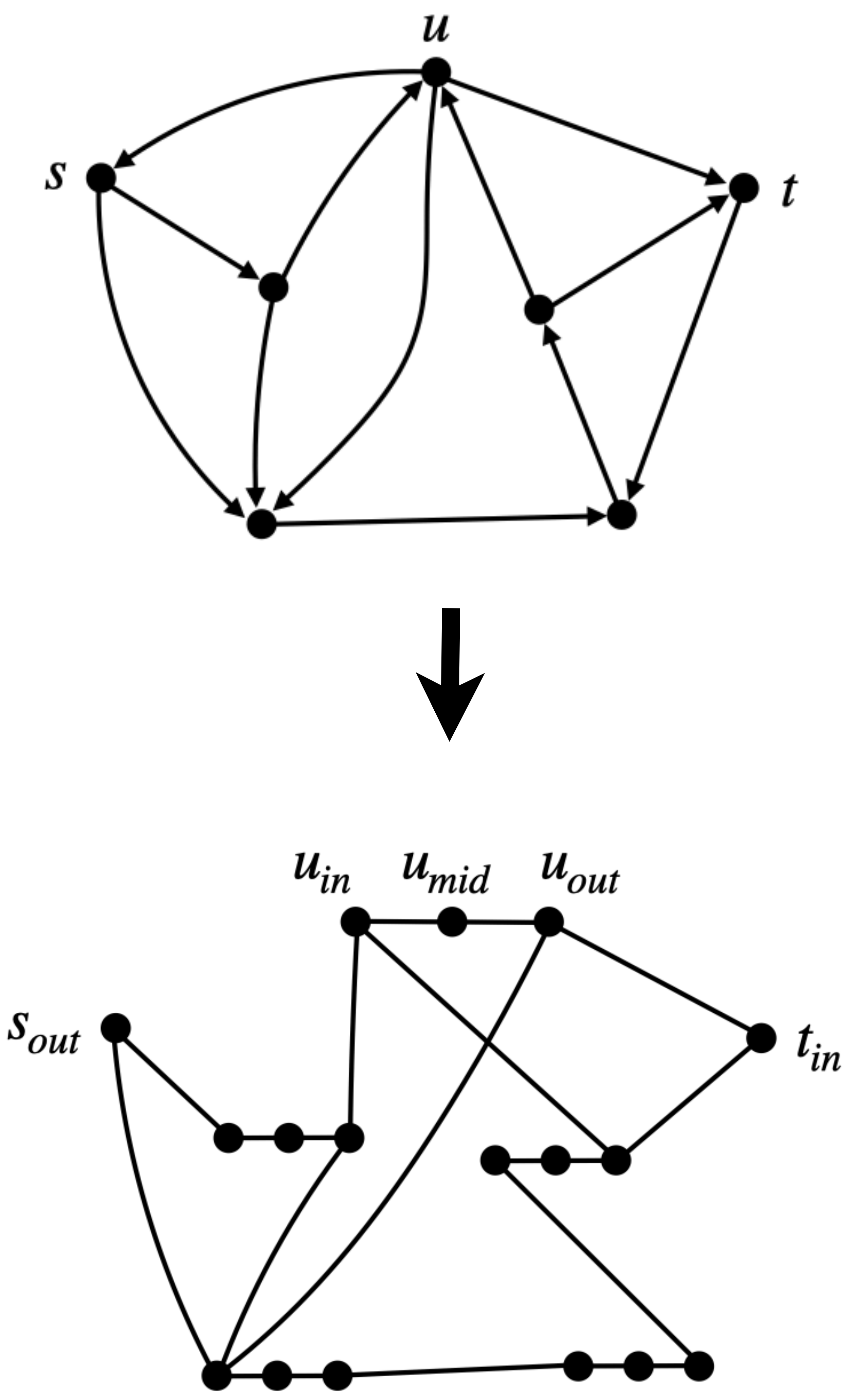
$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$**:**

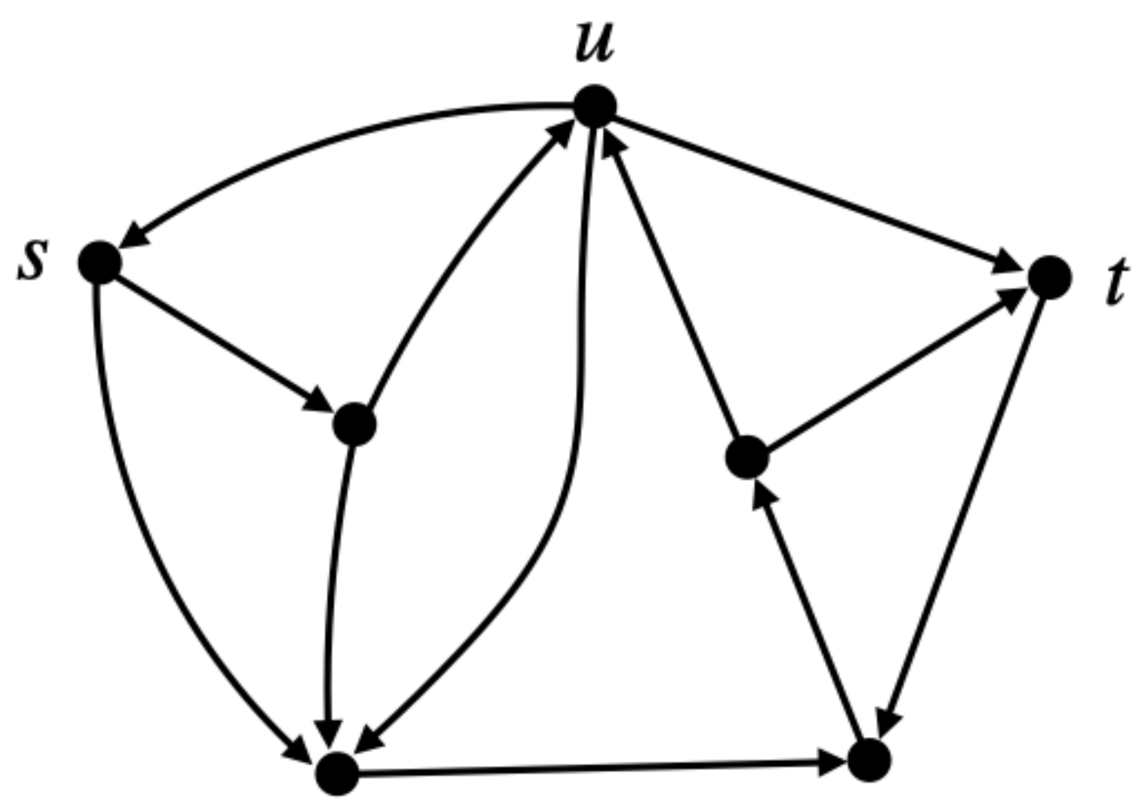
- Vertices of G':

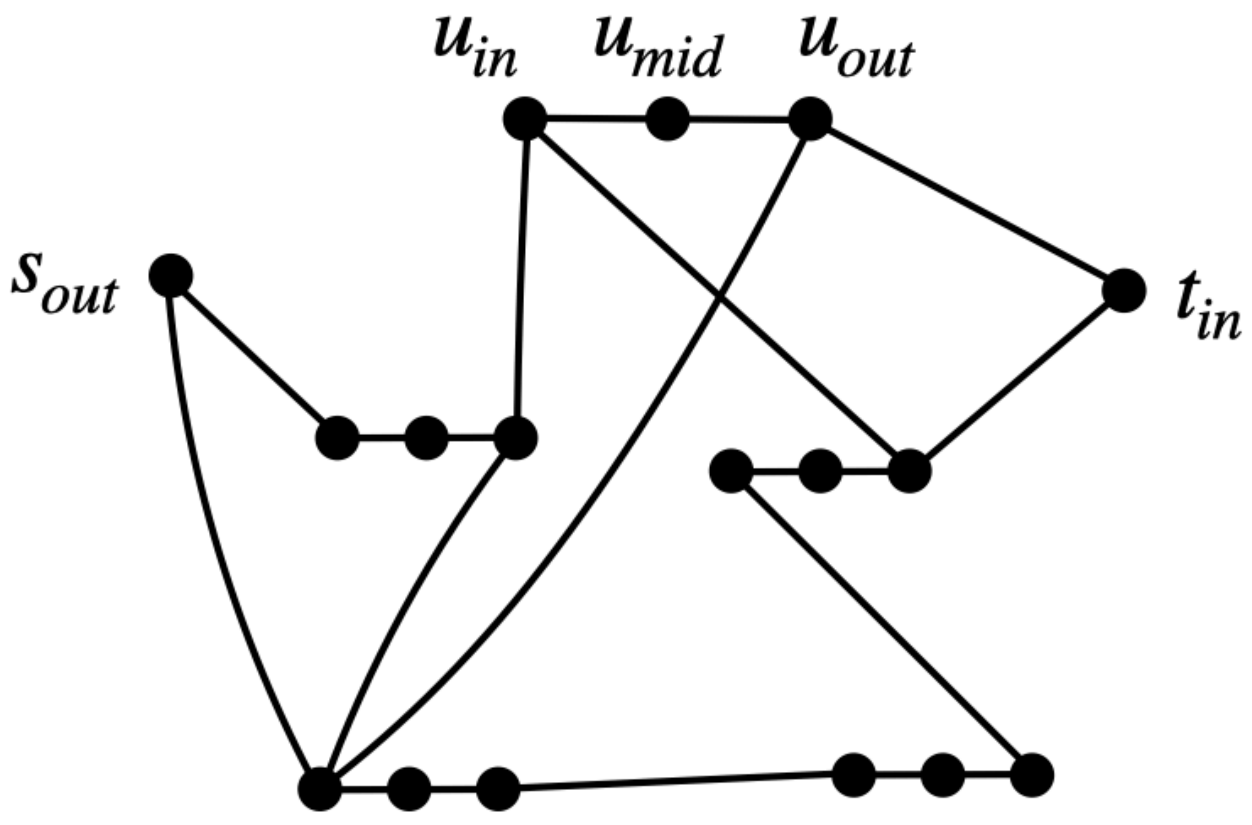


DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:

- Vertices of G':
  - For s add s_{out}, for t add t_{in}.

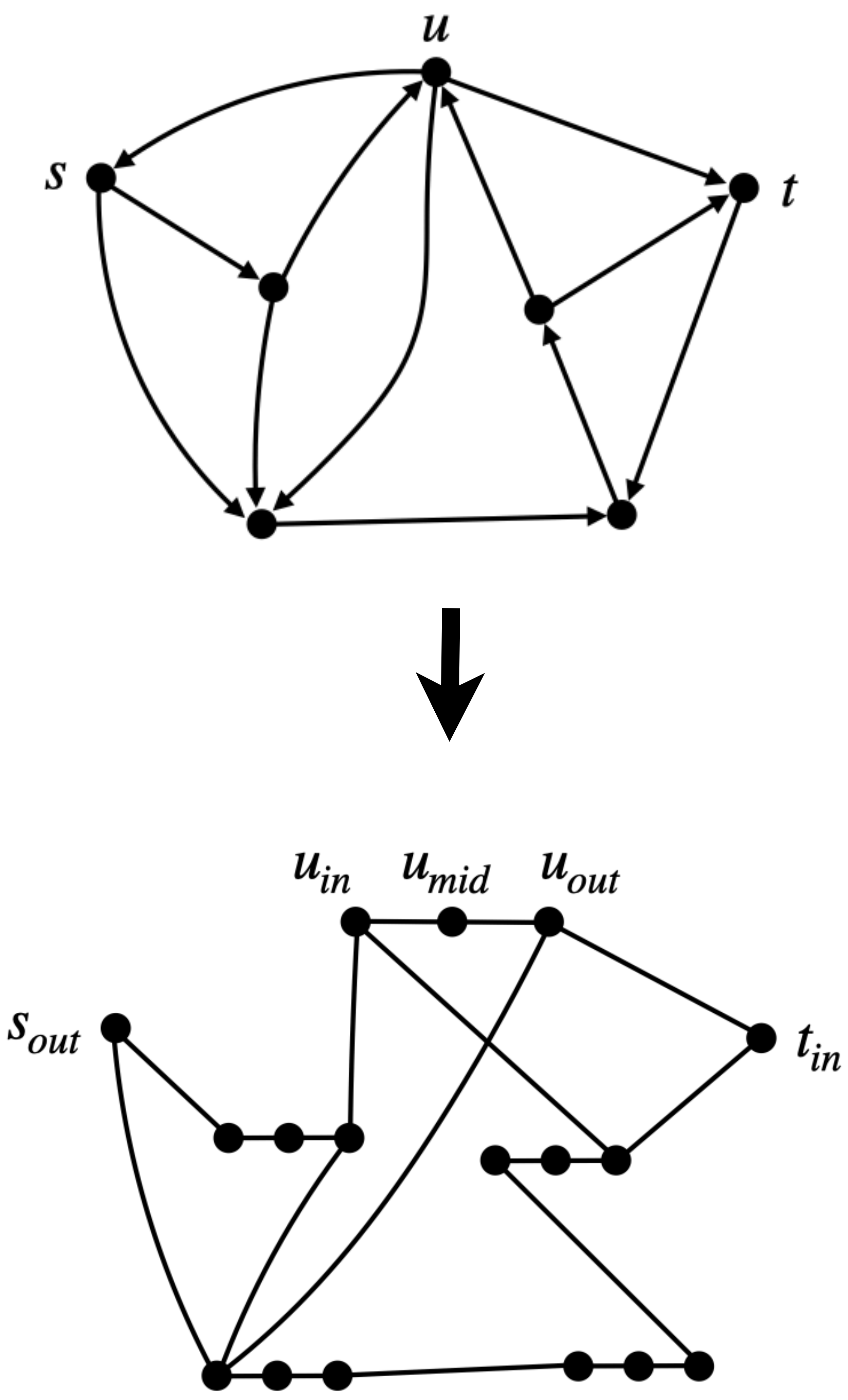




DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:

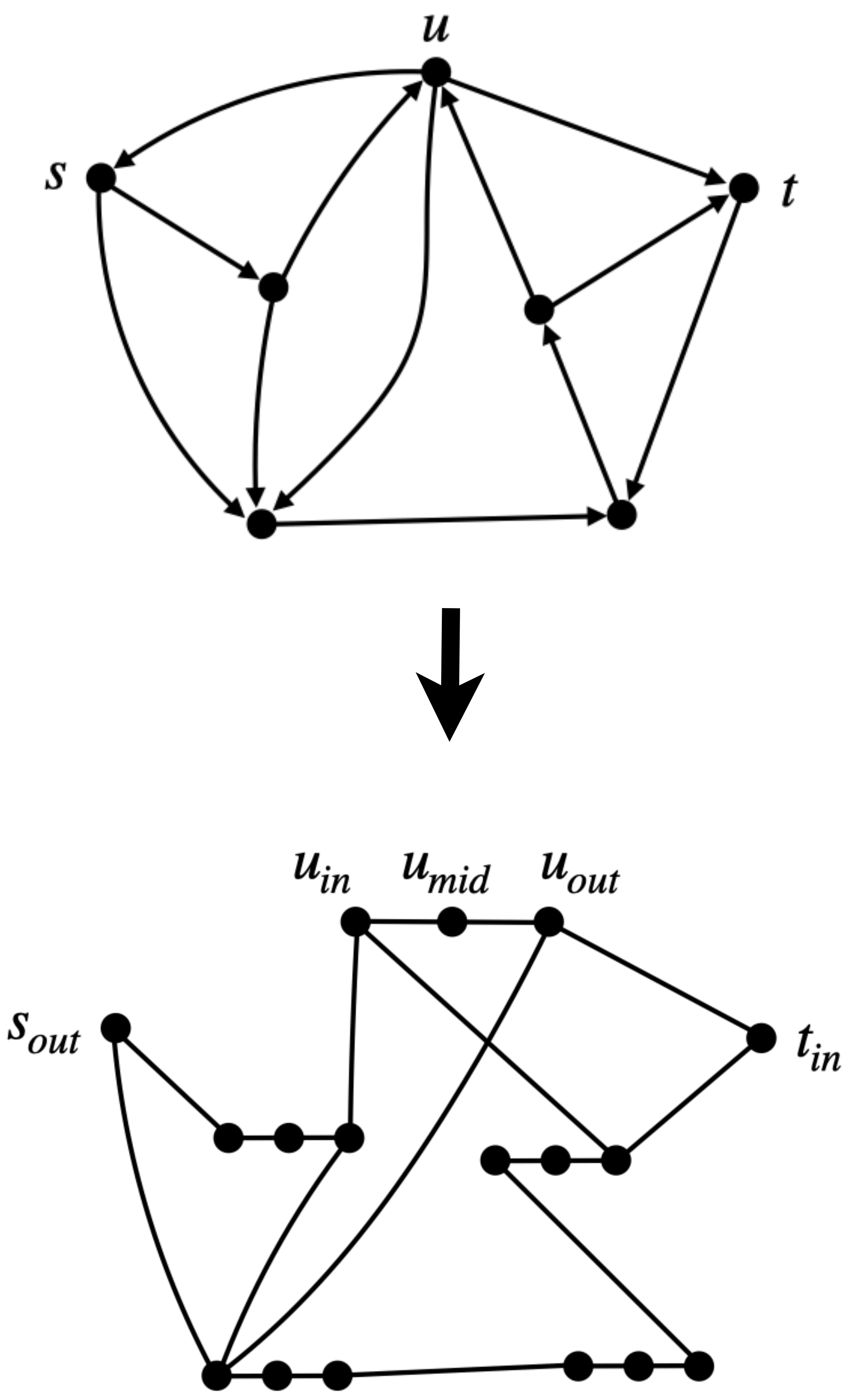
- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:

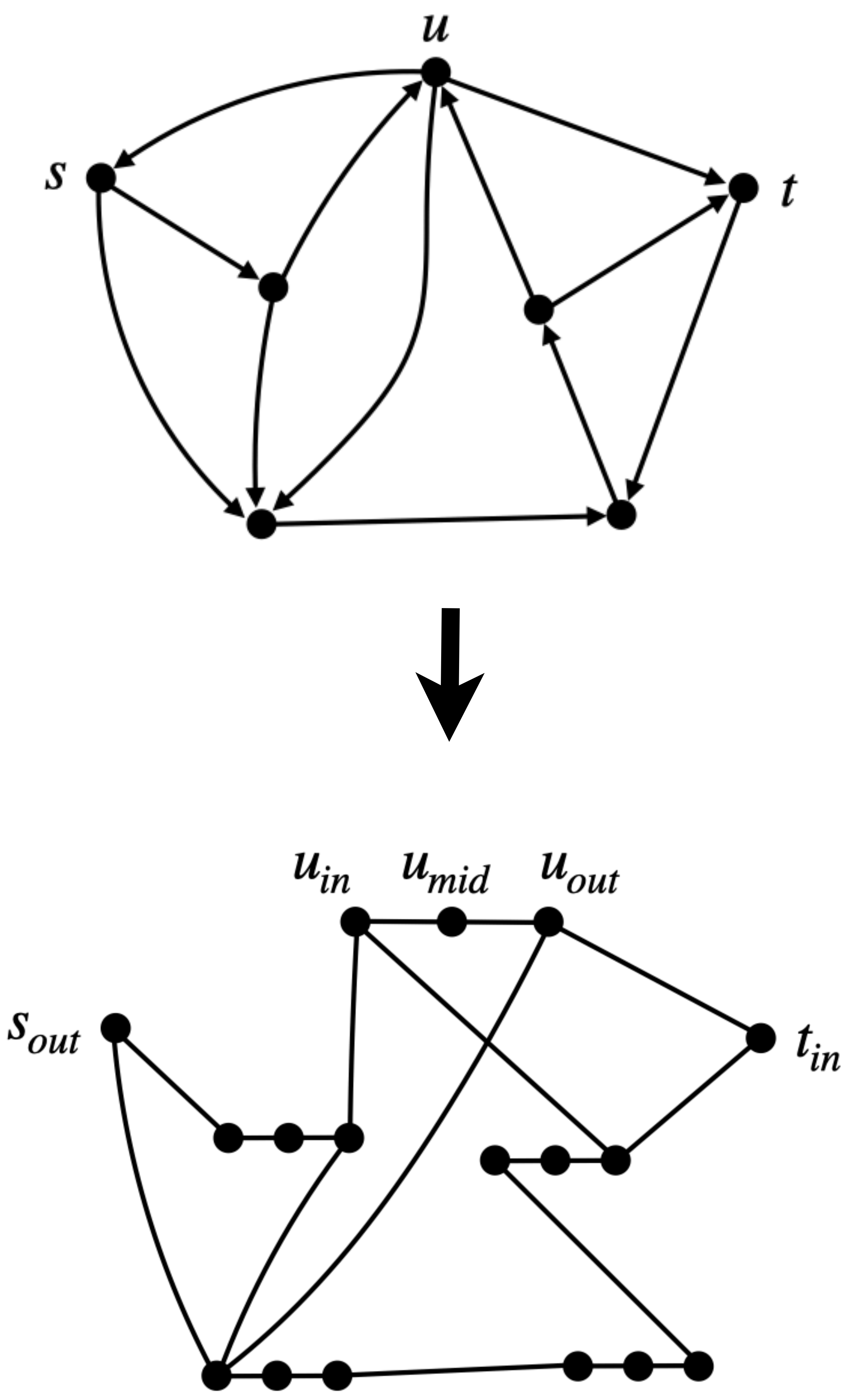
- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t \rangle \rightarrow \langle G', s', t' \rangle$:

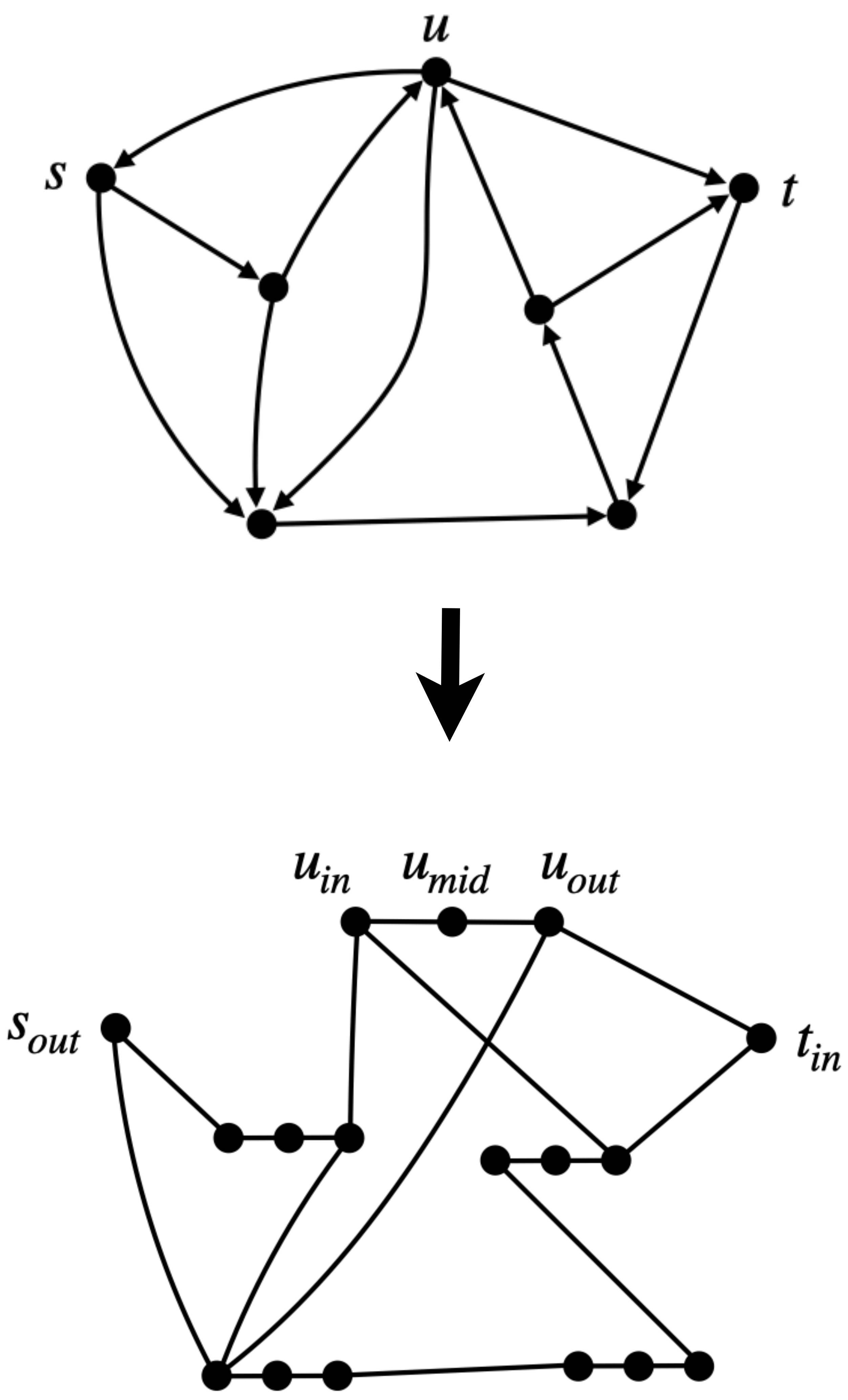
- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':
  - Ignore incoming edges to s and outgoing edges from t.



DirHampath $\leq_p$ *Hampath*

$\langle G, s, t\rangle \rightarrow \langle G', s', t'\rangle$:

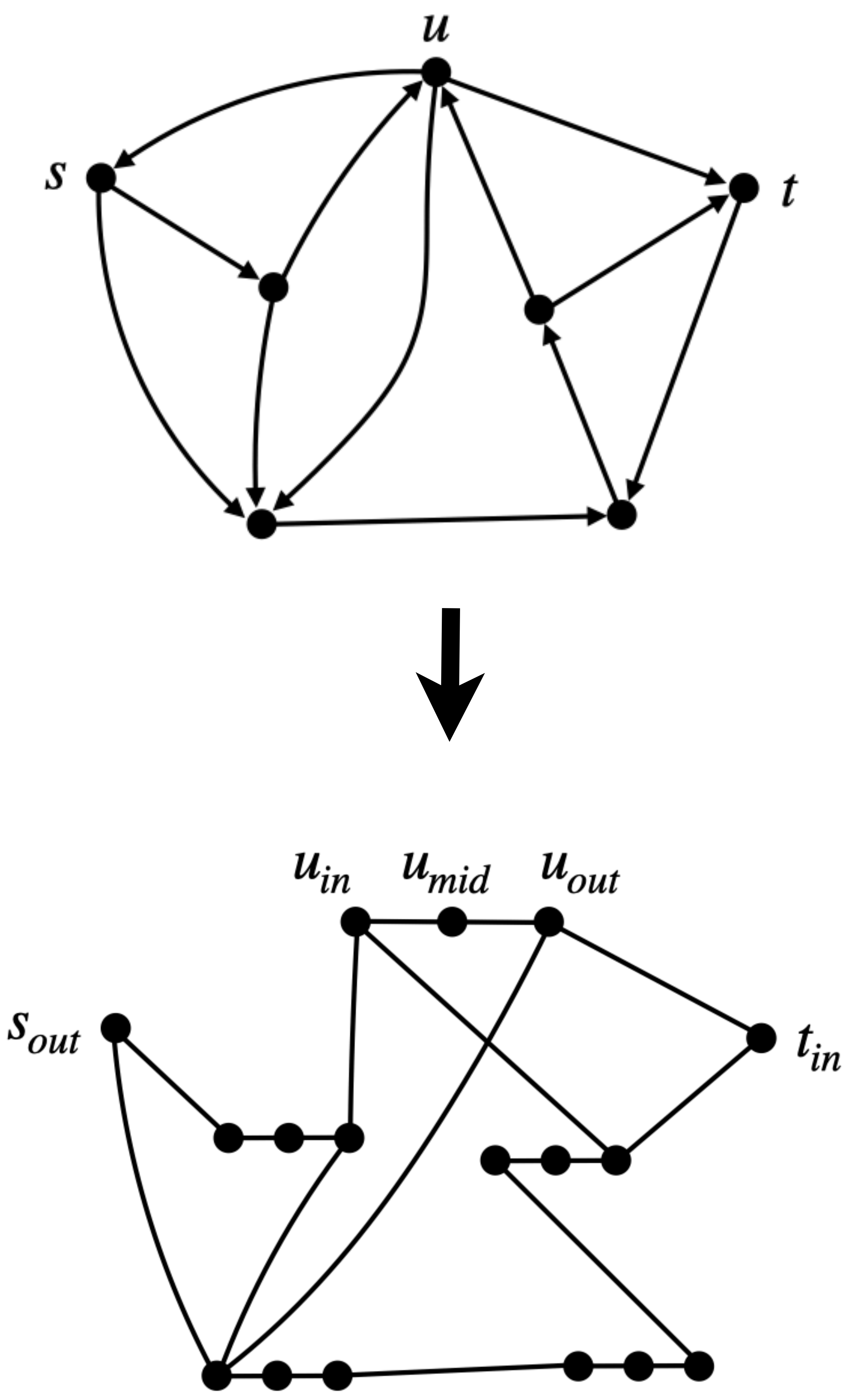
- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':
  - Ignore incoming edges to s and outgoing edges from t.
  - For every edge (u, v) have an edge $\{u_{out}, v_{in}\}$.



DirHampath $\leq_p$ Hampath

$\langle G, s, t\rangle \rightarrow \langle G', s', t'\rangle$:

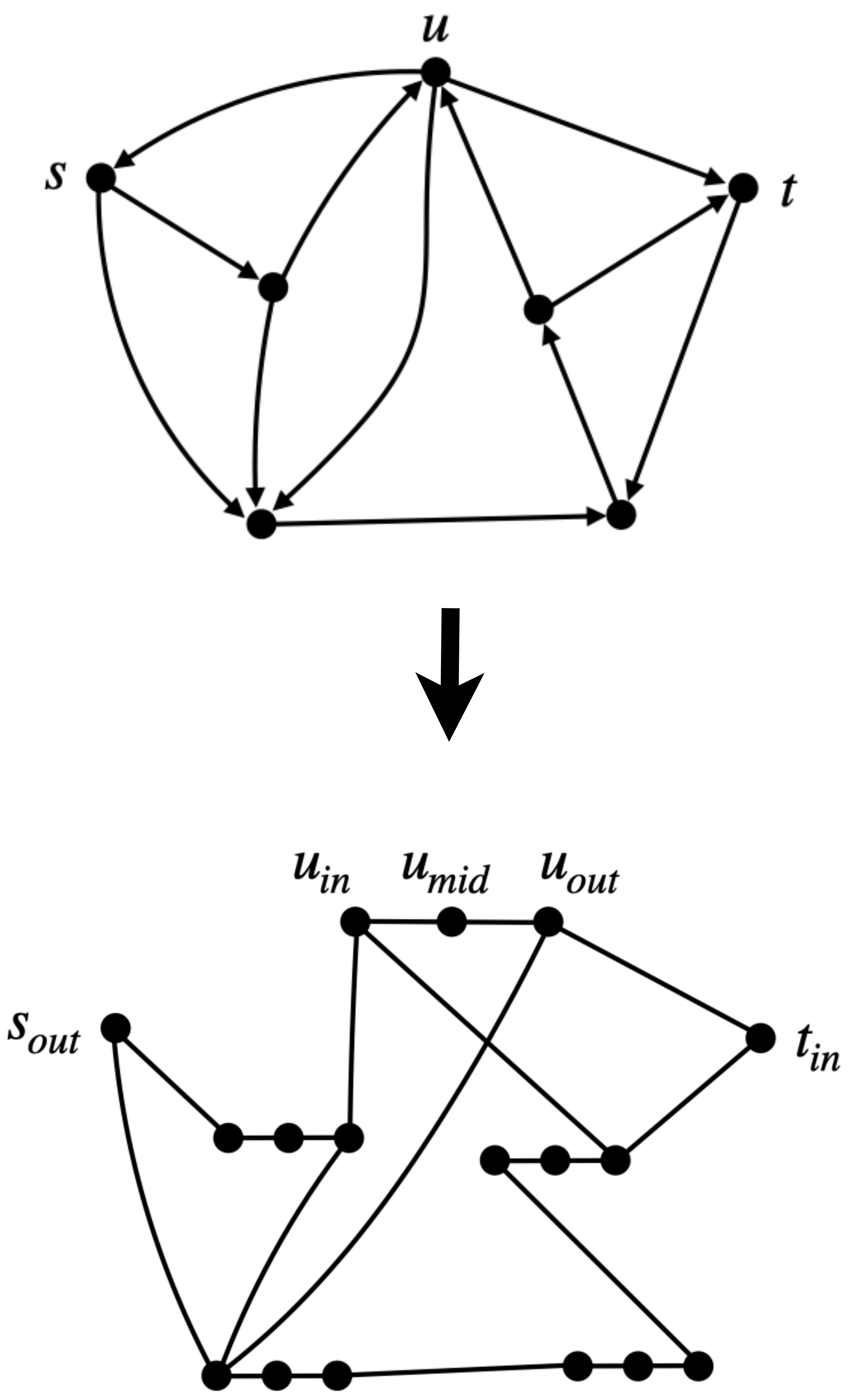
- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':
  - Ignore incoming edges to s and outgoing edges from t.
  - For every edge (u, v) have an edge $\{u_{out}, v_{in}\}$.
  - Edges $\{u_{in}, u_{mid}\}$ and $\{u_{mid}, u_{out}\}$.



DirHampath $\leq_p$ Hampath

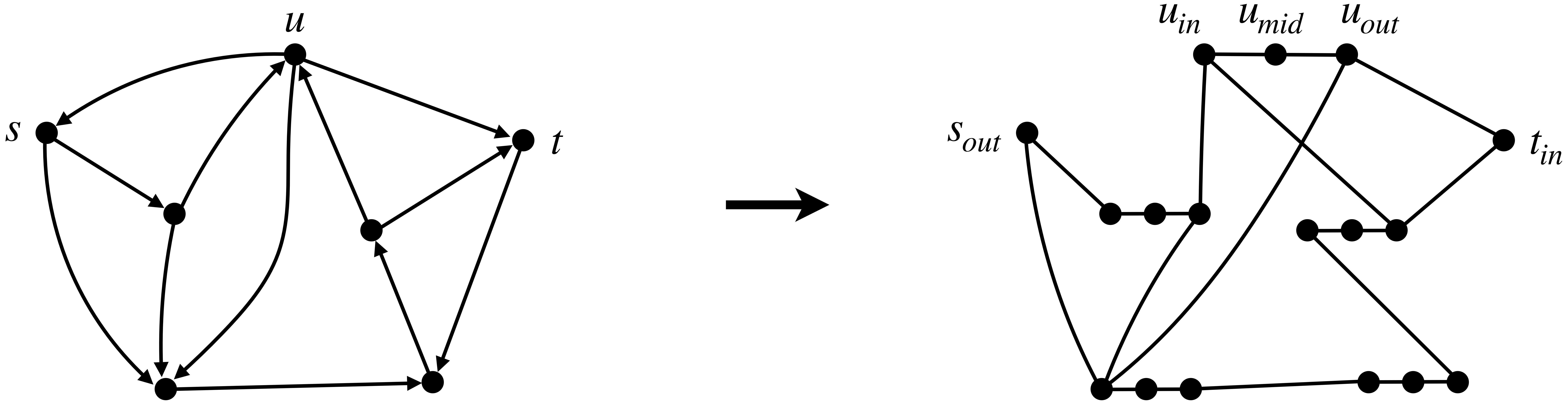
$\langle G, s, t \rangle \to \langle G', s', t' \rangle$:

- Vertices of G':
  - For s add s_{out}, for t add t_{in}.
  - For any other vertex u add 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':
  - Ignore incoming edges to s and outgoing edges from t.
  - For every edge (u, v) have an edge $\{u_{out}, v_{in}\}$.
  - Edges $\{u_{in}, u_{mid}\}$ and $\{u_{mid}, u_{out}\}$.
- $s' = s_{out}$, $t' = t_{in}$.



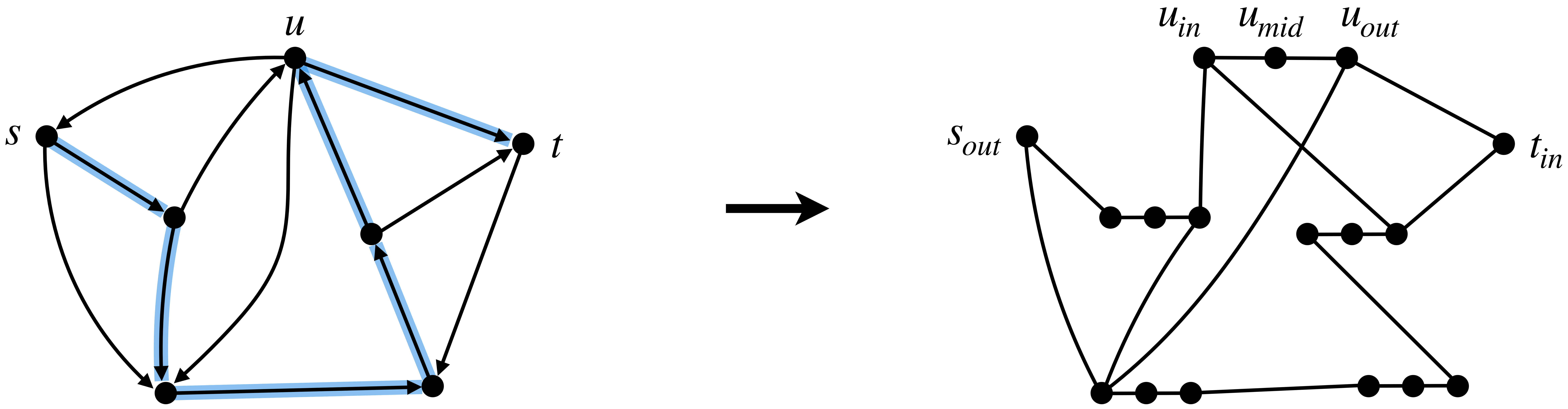
DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$):



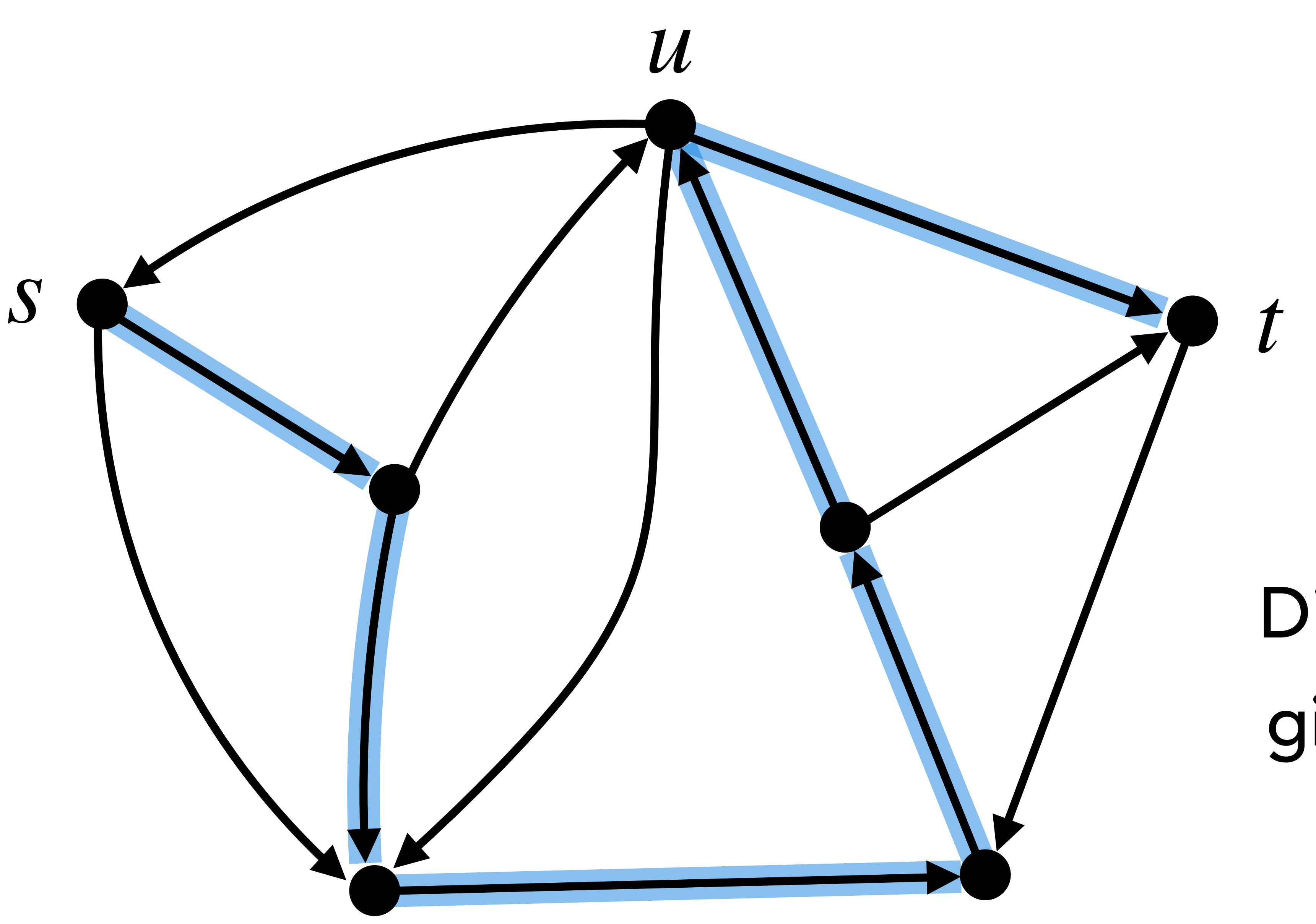
$DirHampath \leq_p Hampath$

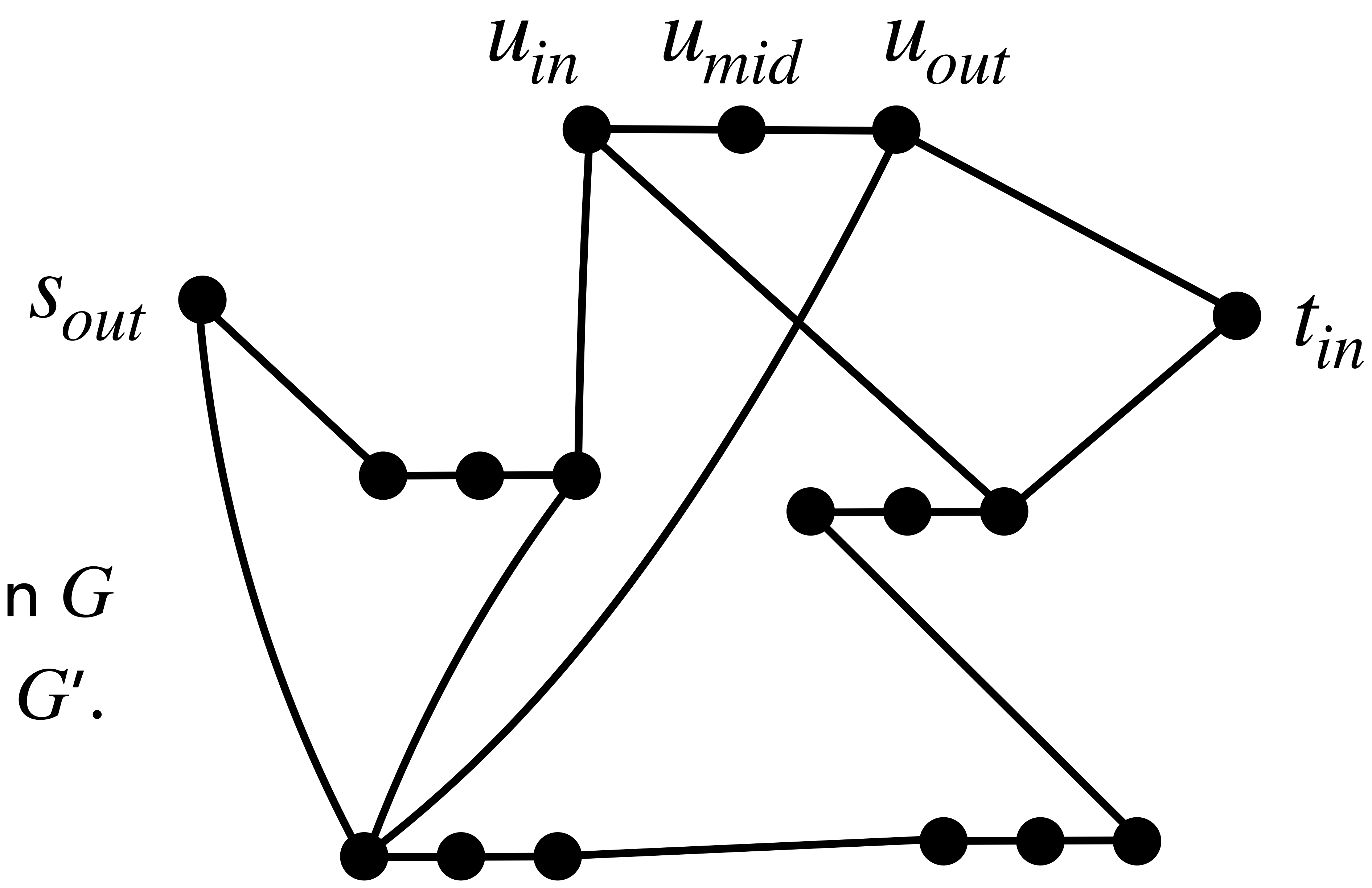
Correctness of reduction ($\Longrightarrow$):



DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longrightarrow)$**:**

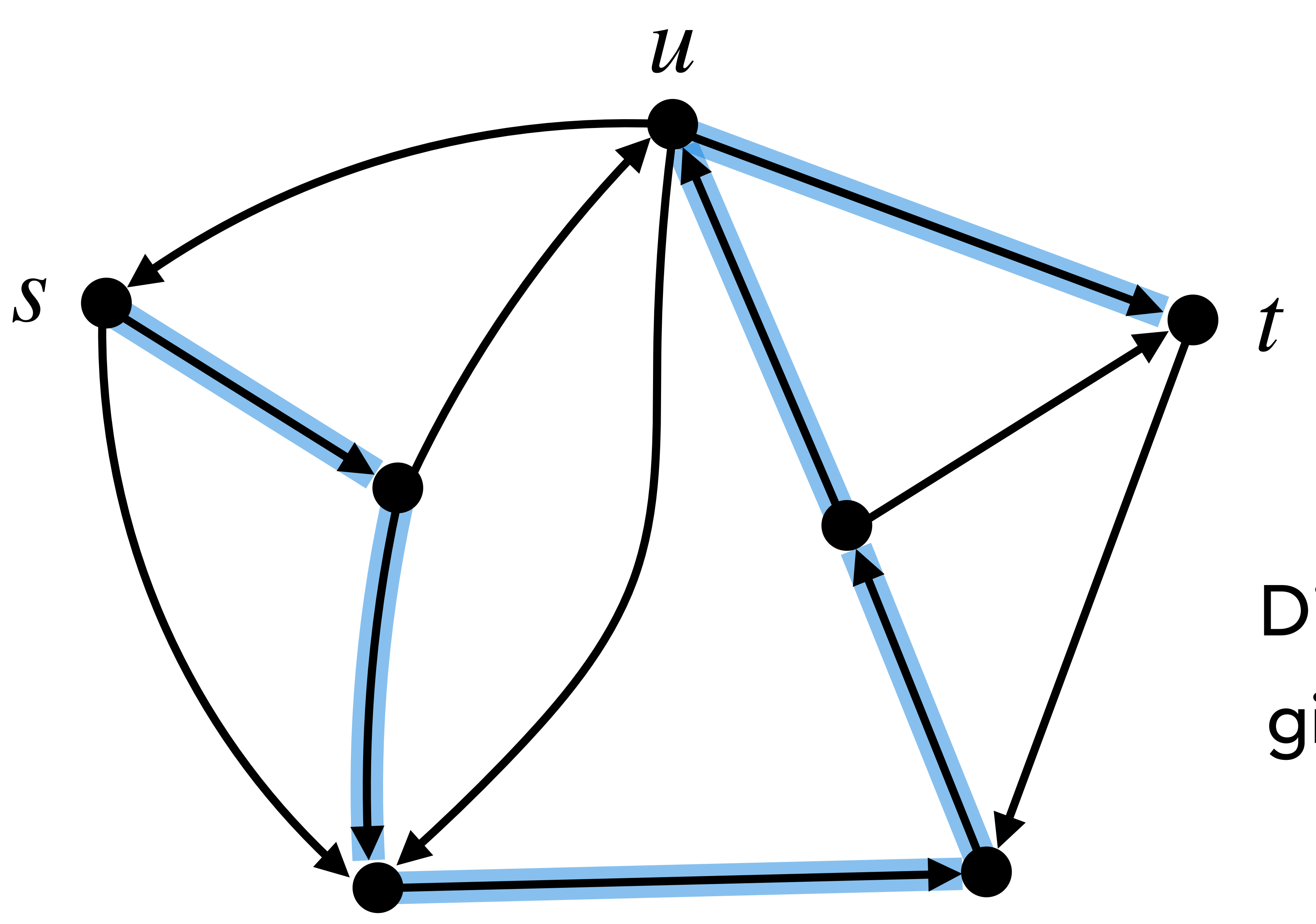




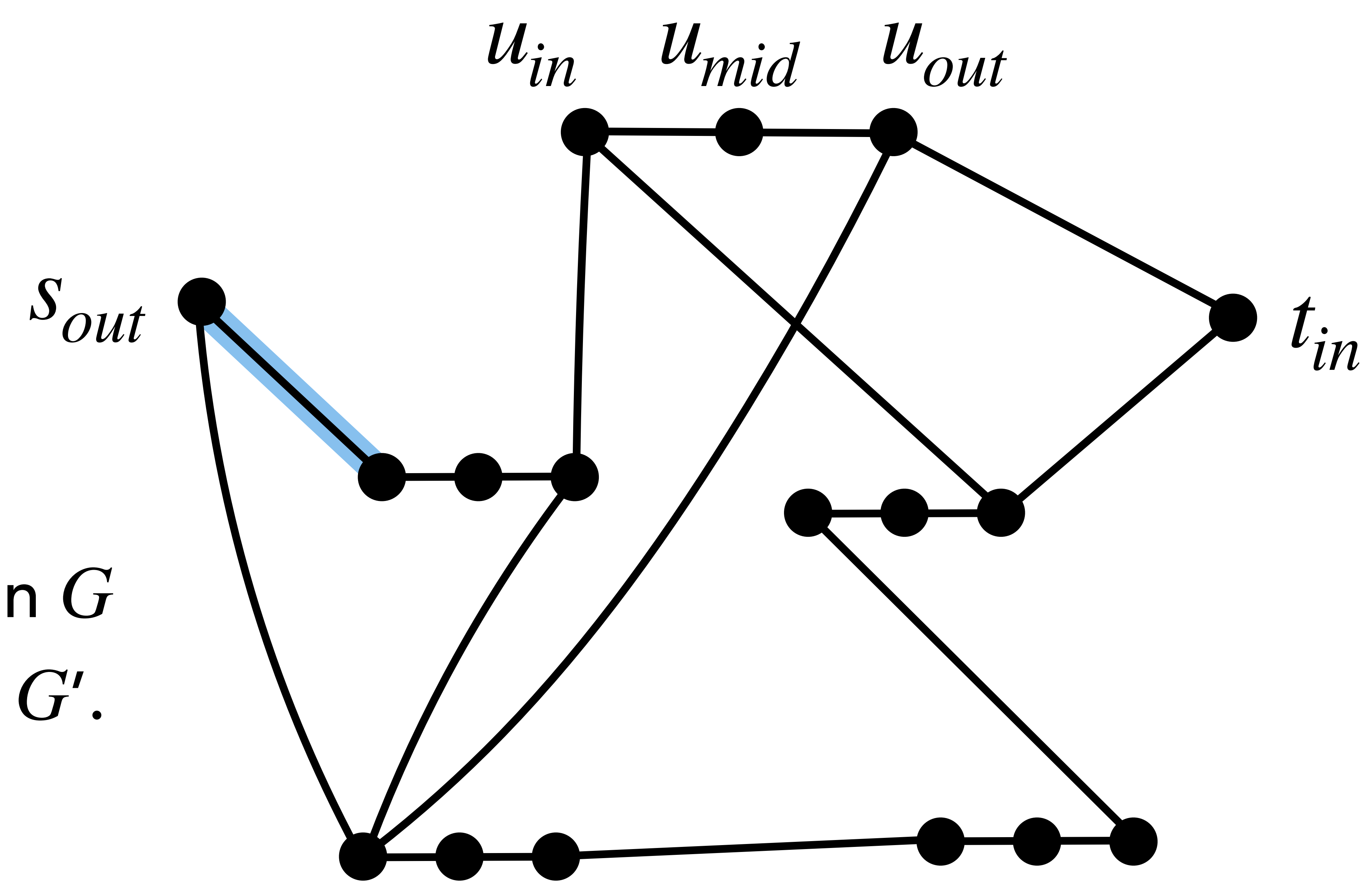
Directed hamiltonian path in G gives a hamiltonian path in G'.

DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longrightarrow)$**:**

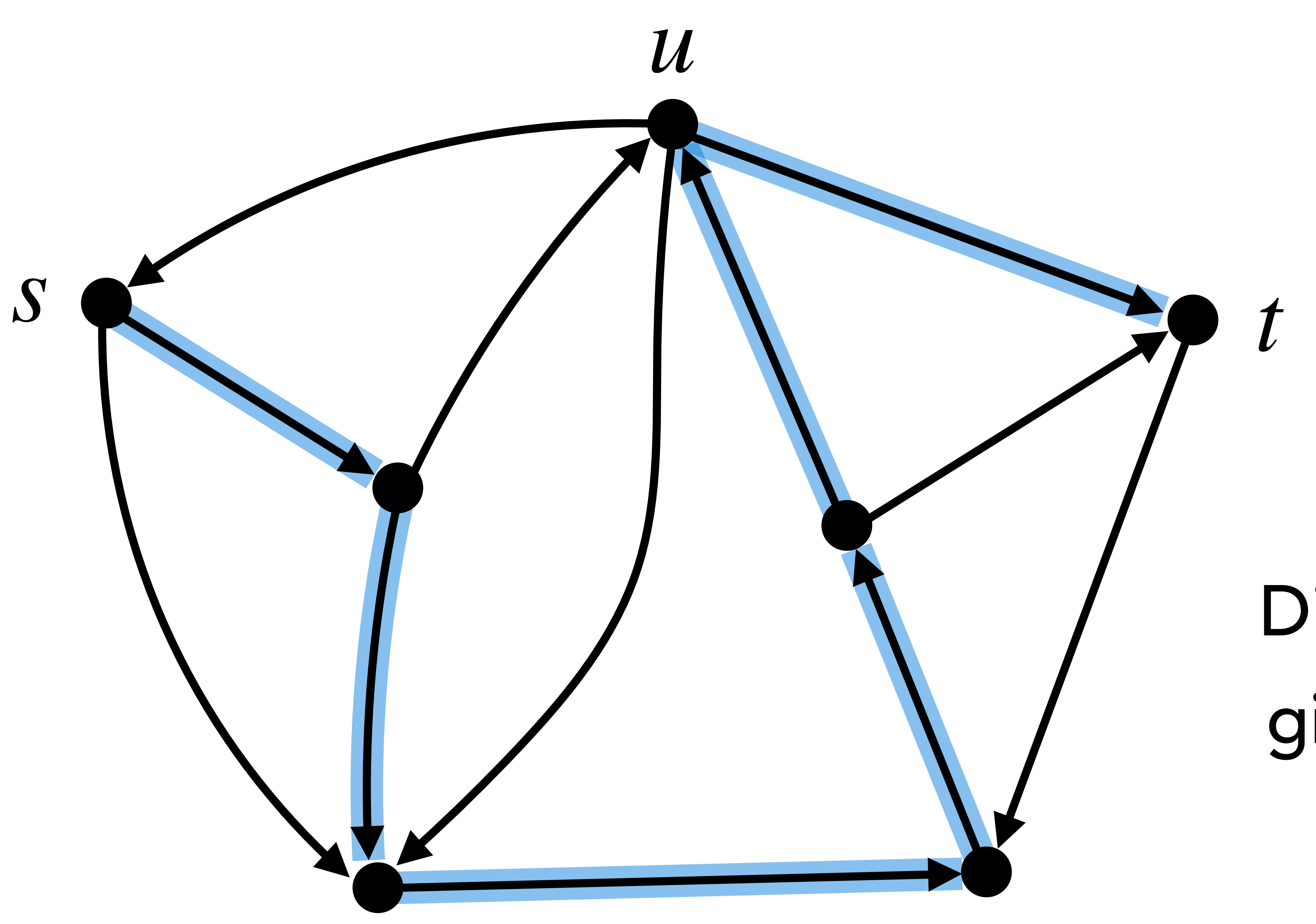


Directed hamiltonian path in G gives a hamiltonian path in G'.

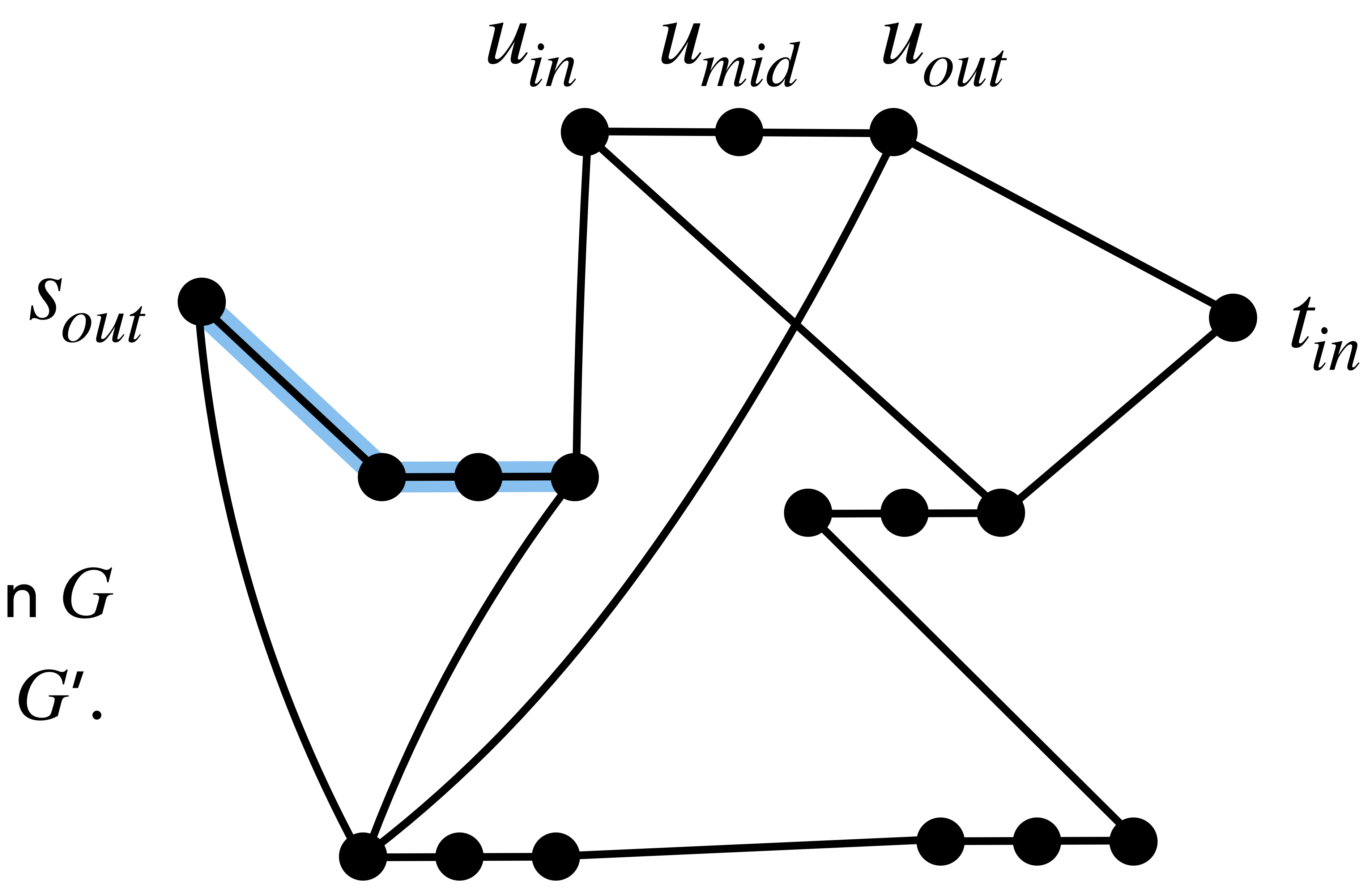


$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longrightarrow$):

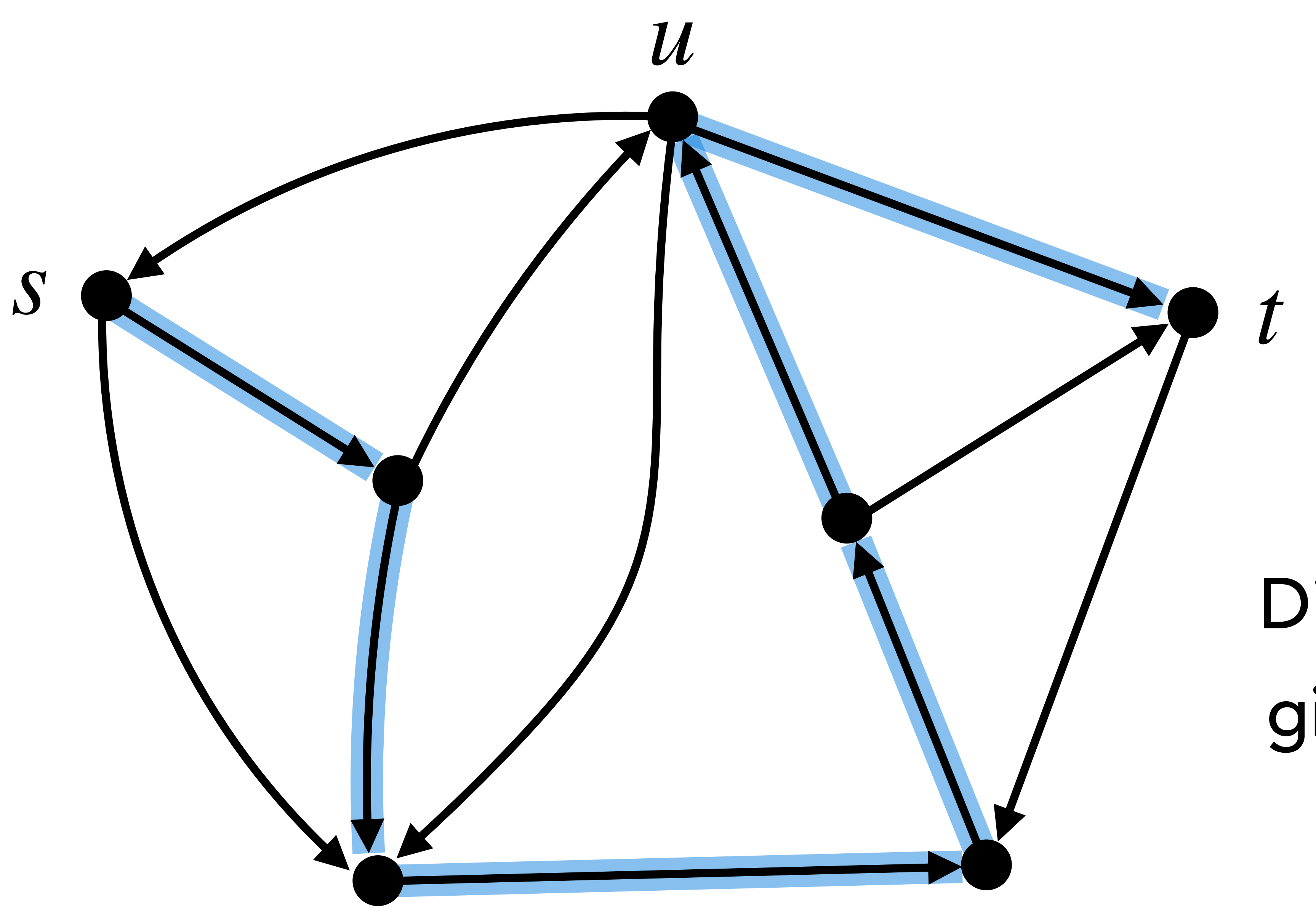


Directed hamiltonian path in G gives a hamiltonian path in G'.

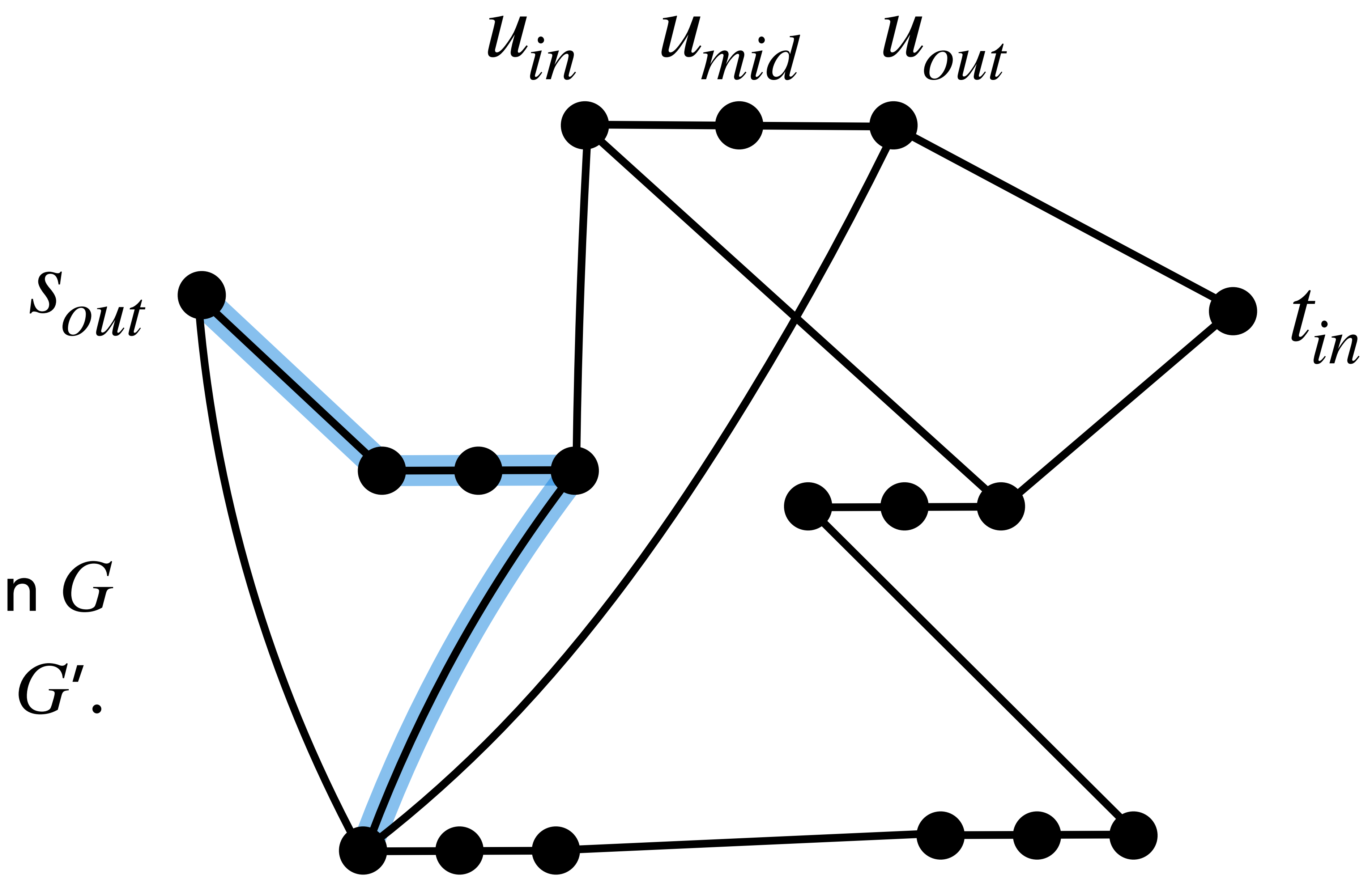


$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longrightarrow$):

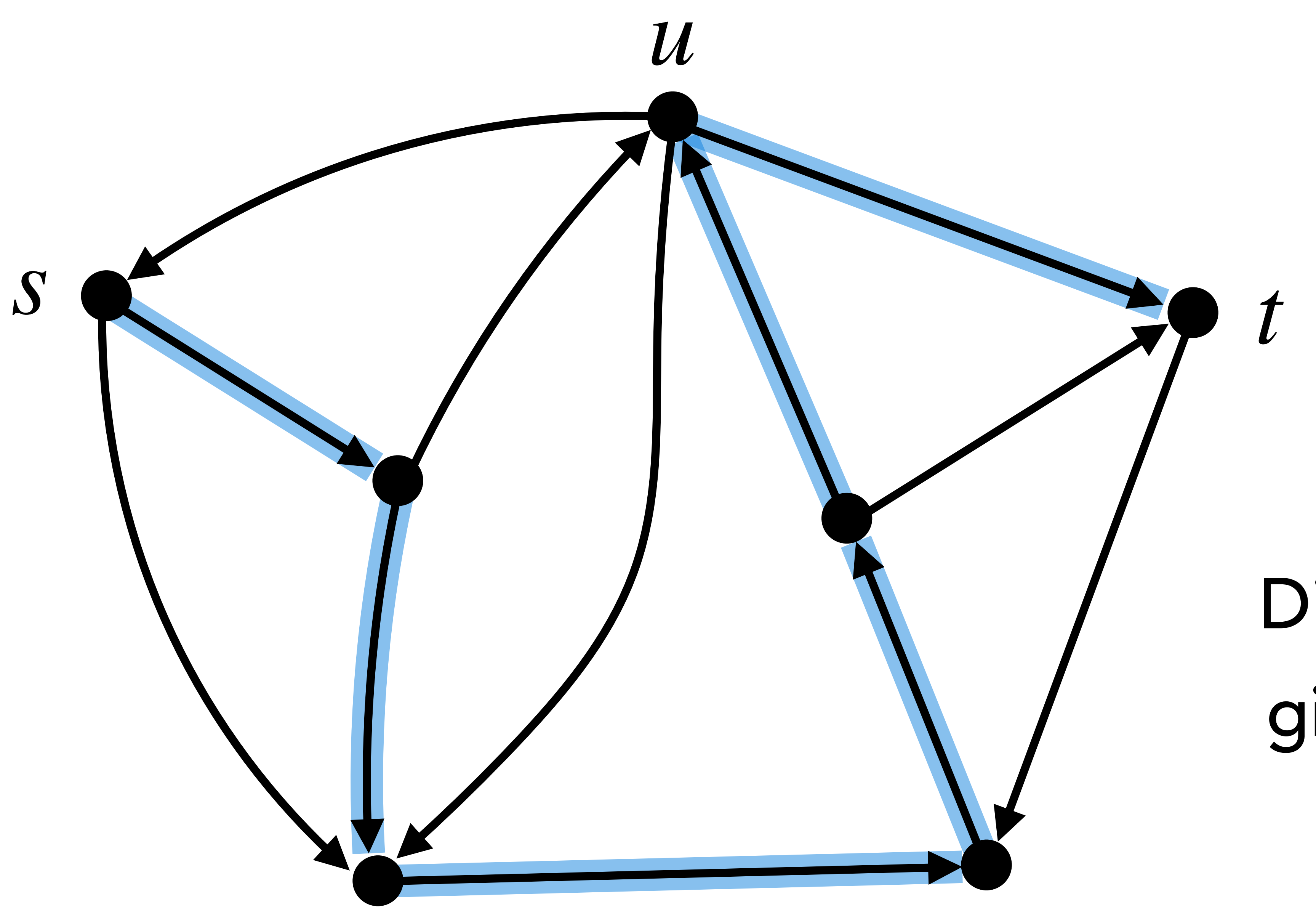


Directed hamiltonian path in G gives a hamiltonian path in G'.

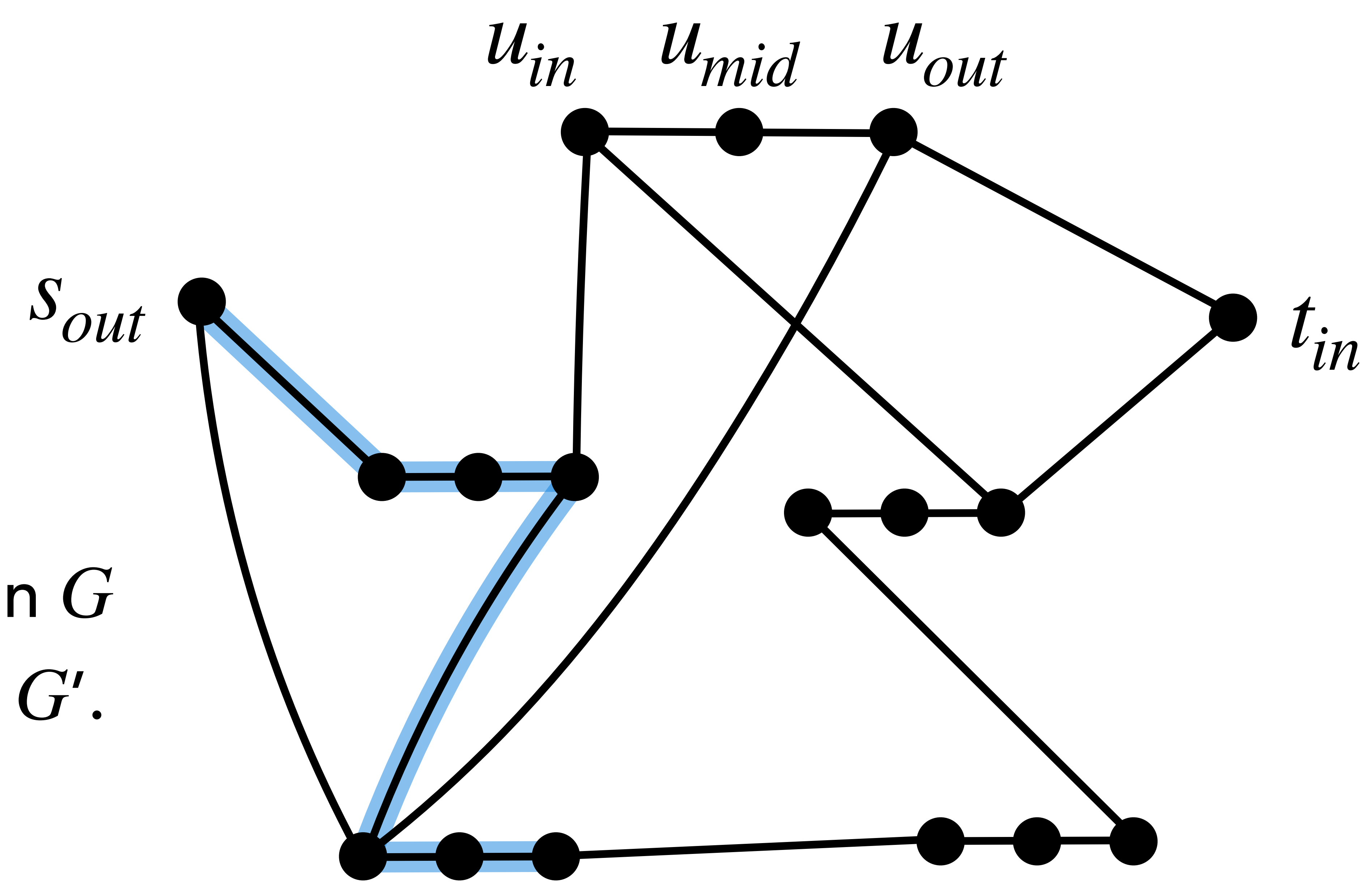


$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longrightarrow$):

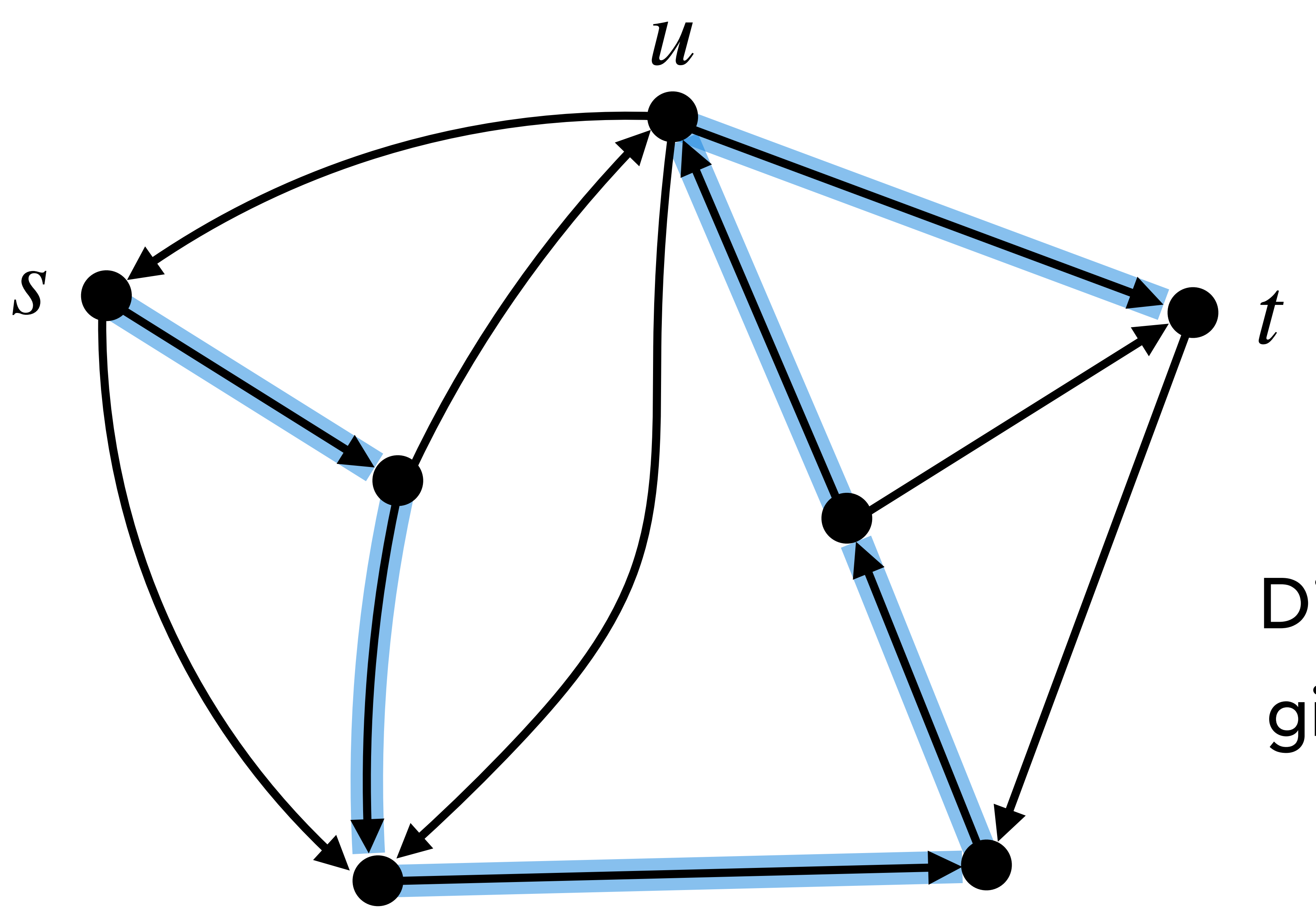


Directed hamiltonian path in G gives a hamiltonian path in G'.

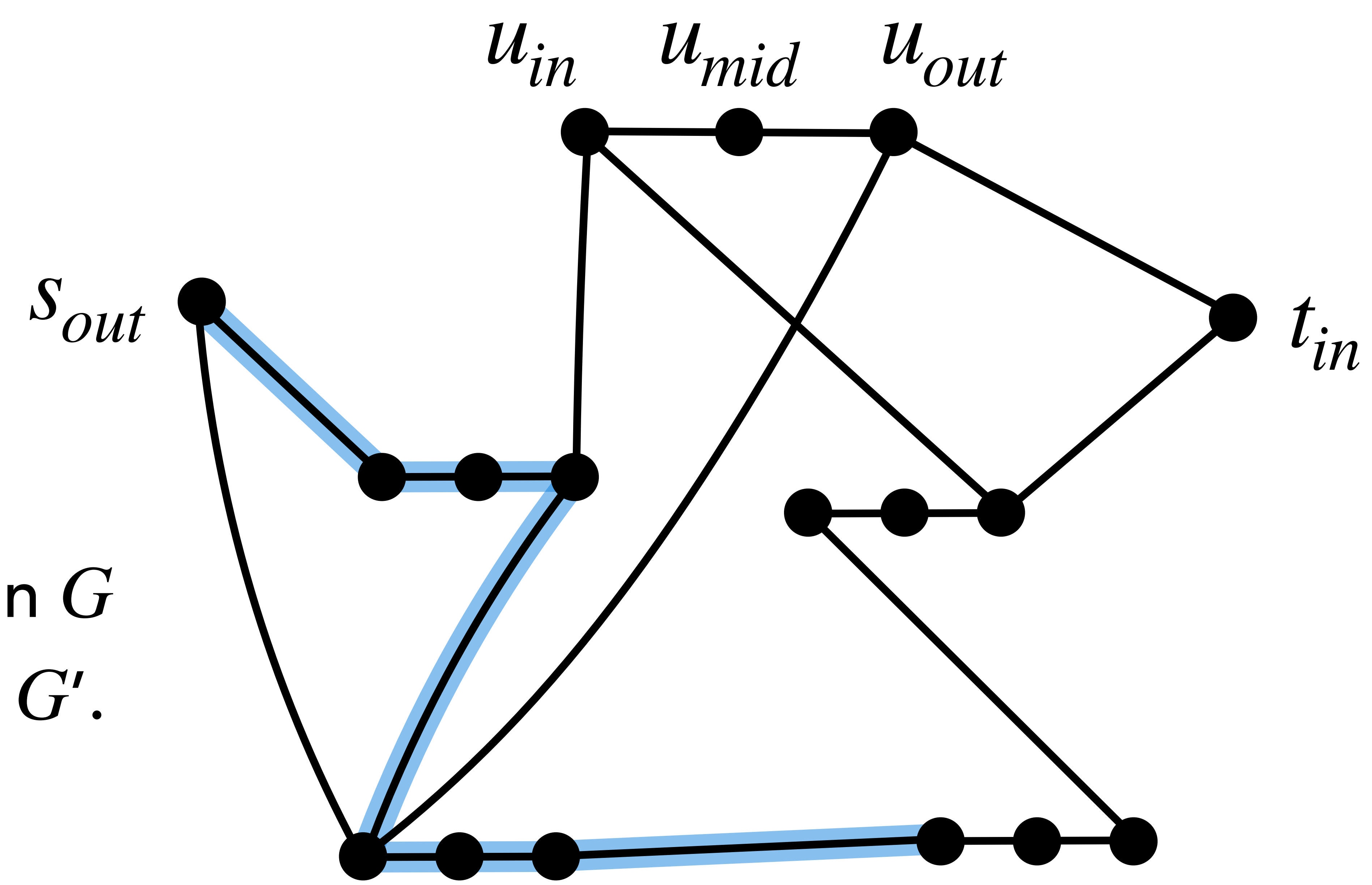


DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$):

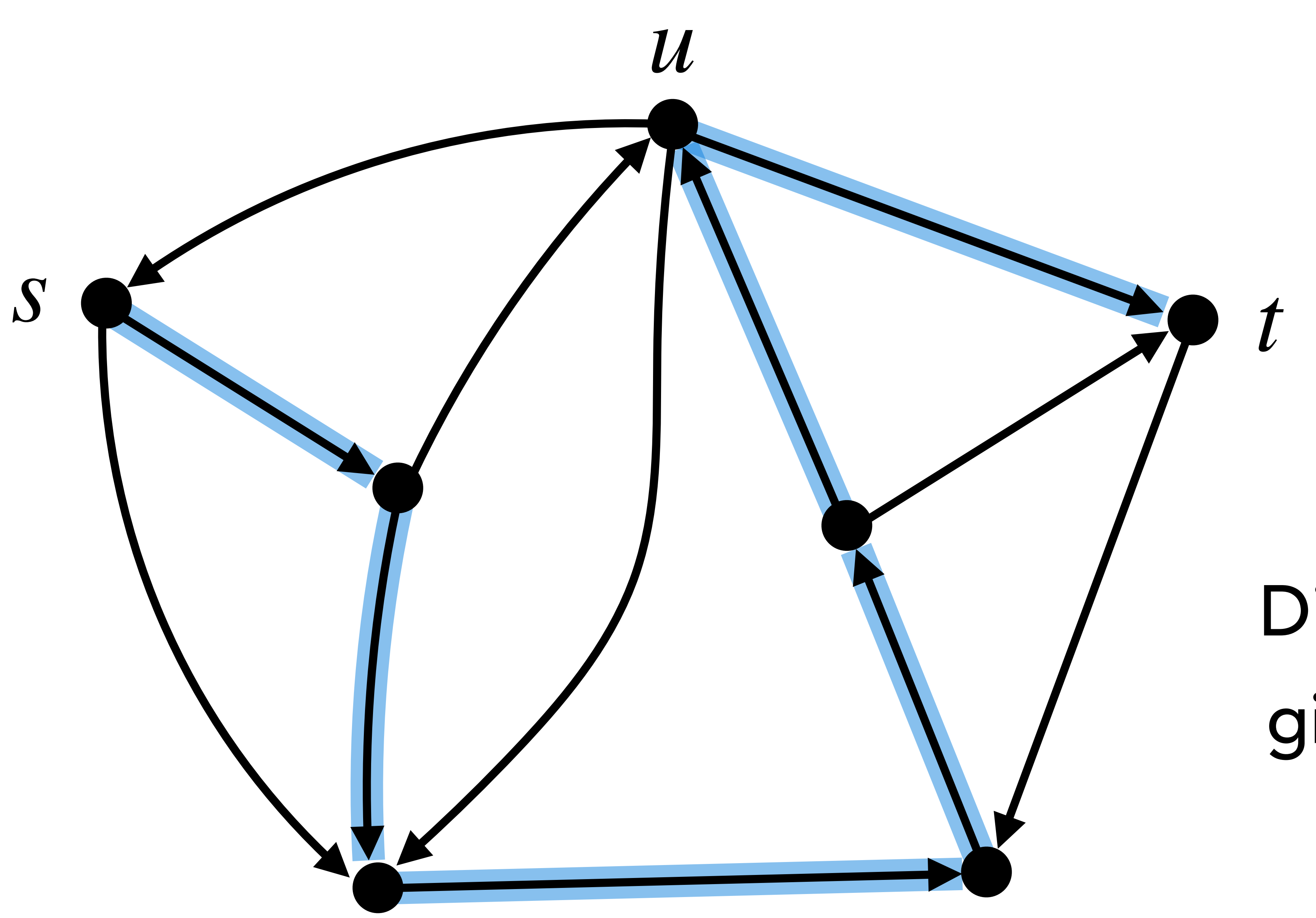


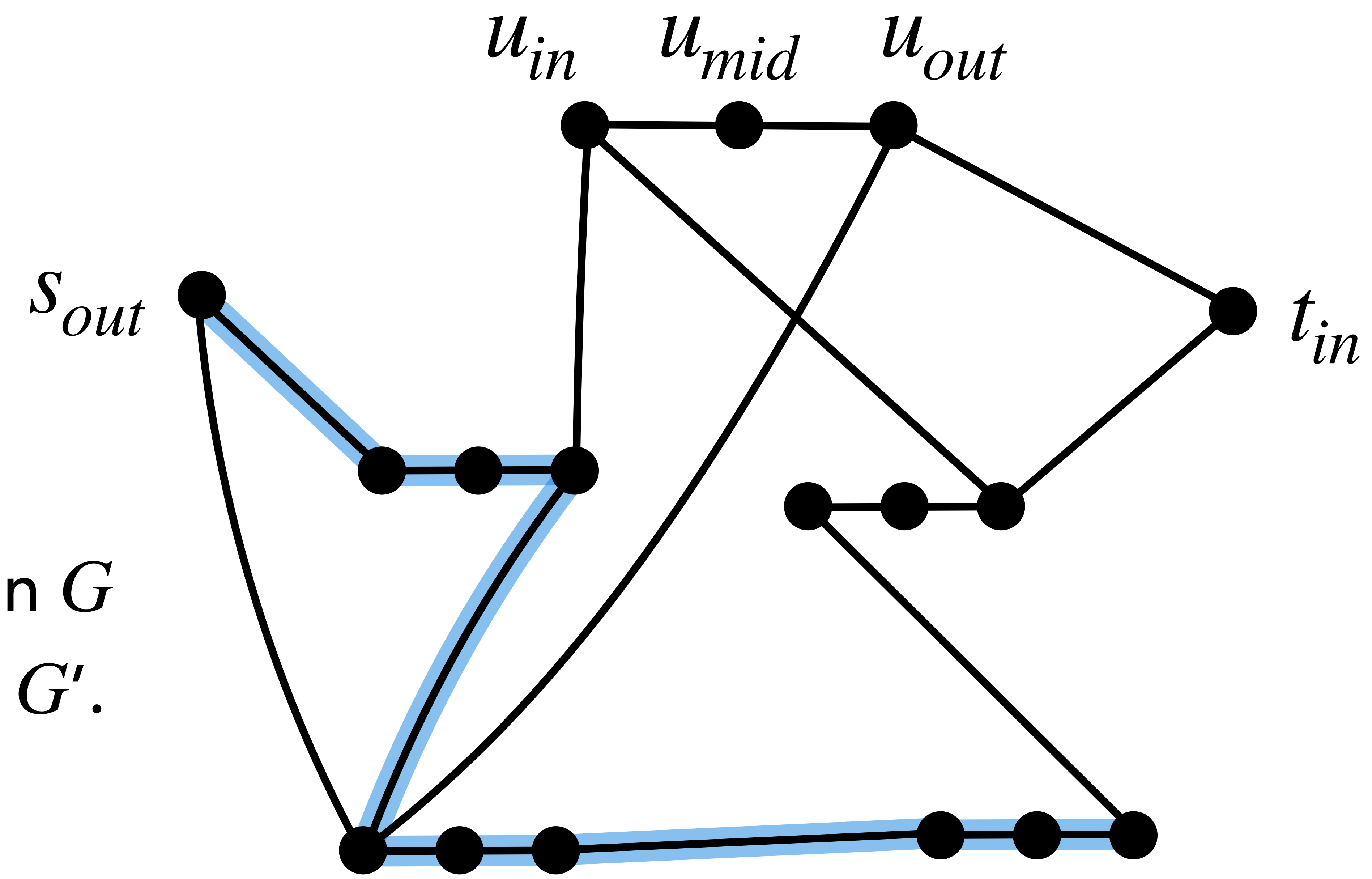
Directed hamiltonian path in G gives a hamiltonian path in G'.



DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$):

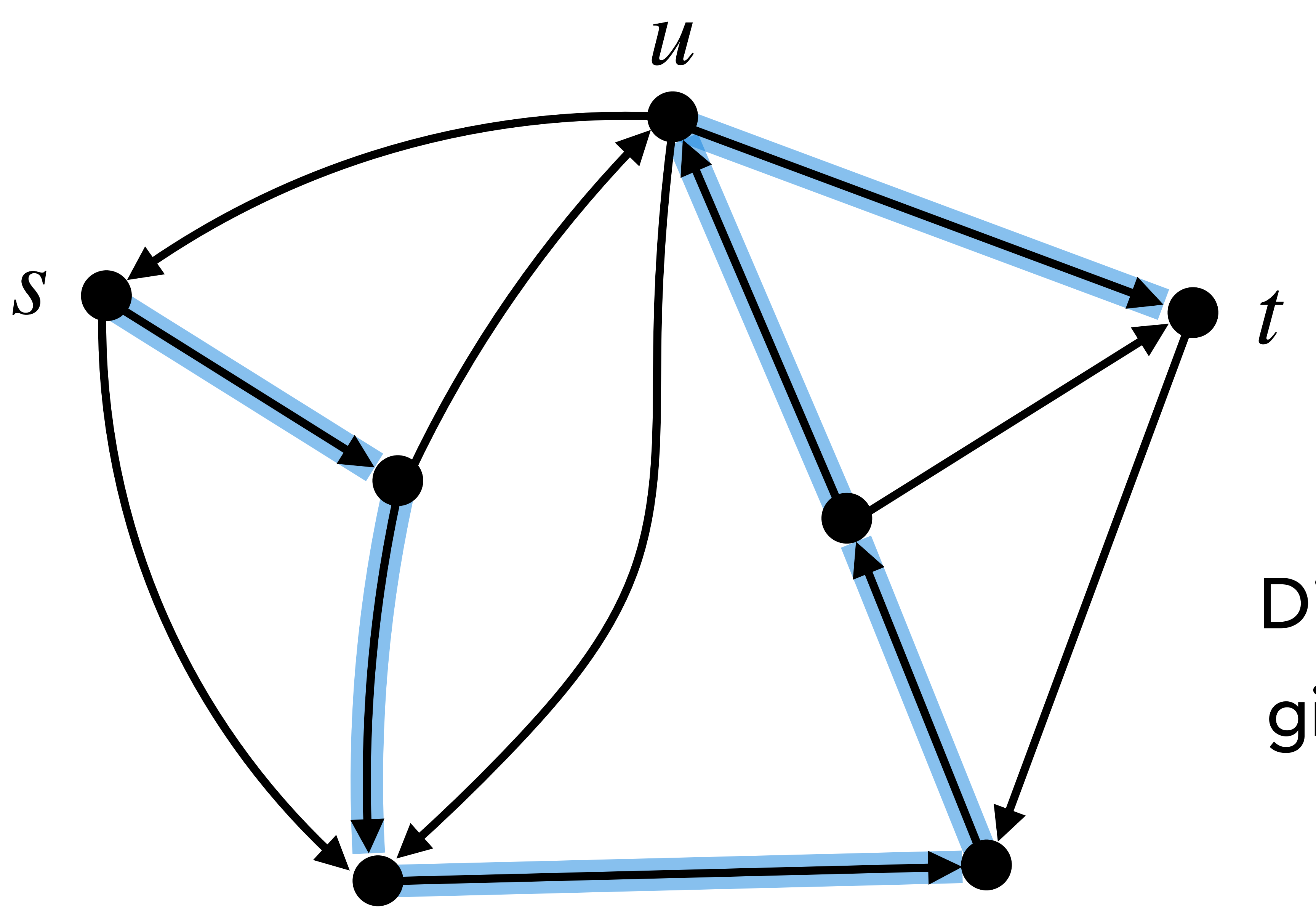




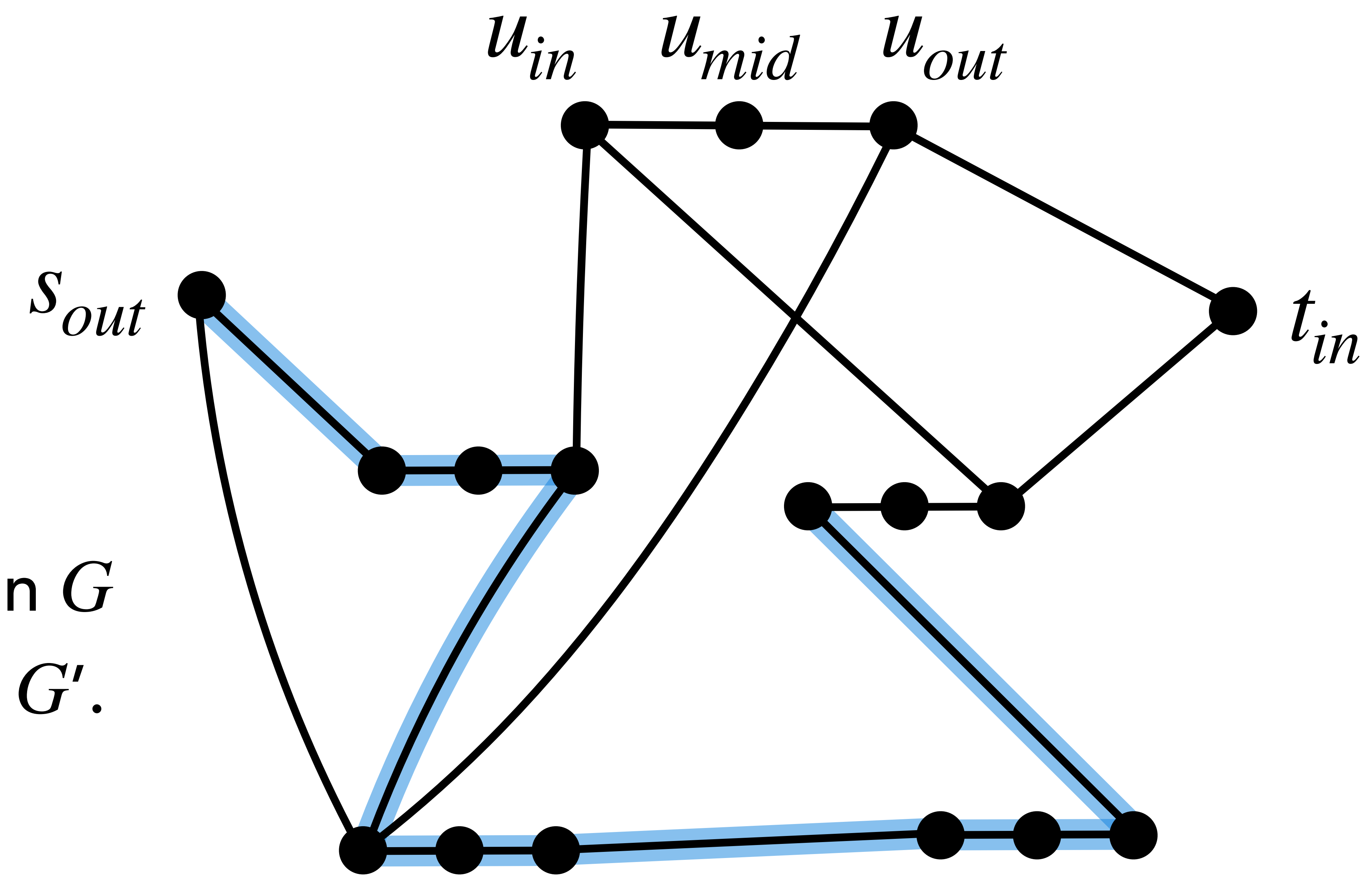
Directed hamiltonian path in G gives a hamiltonian path in G'.

DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$)**:**

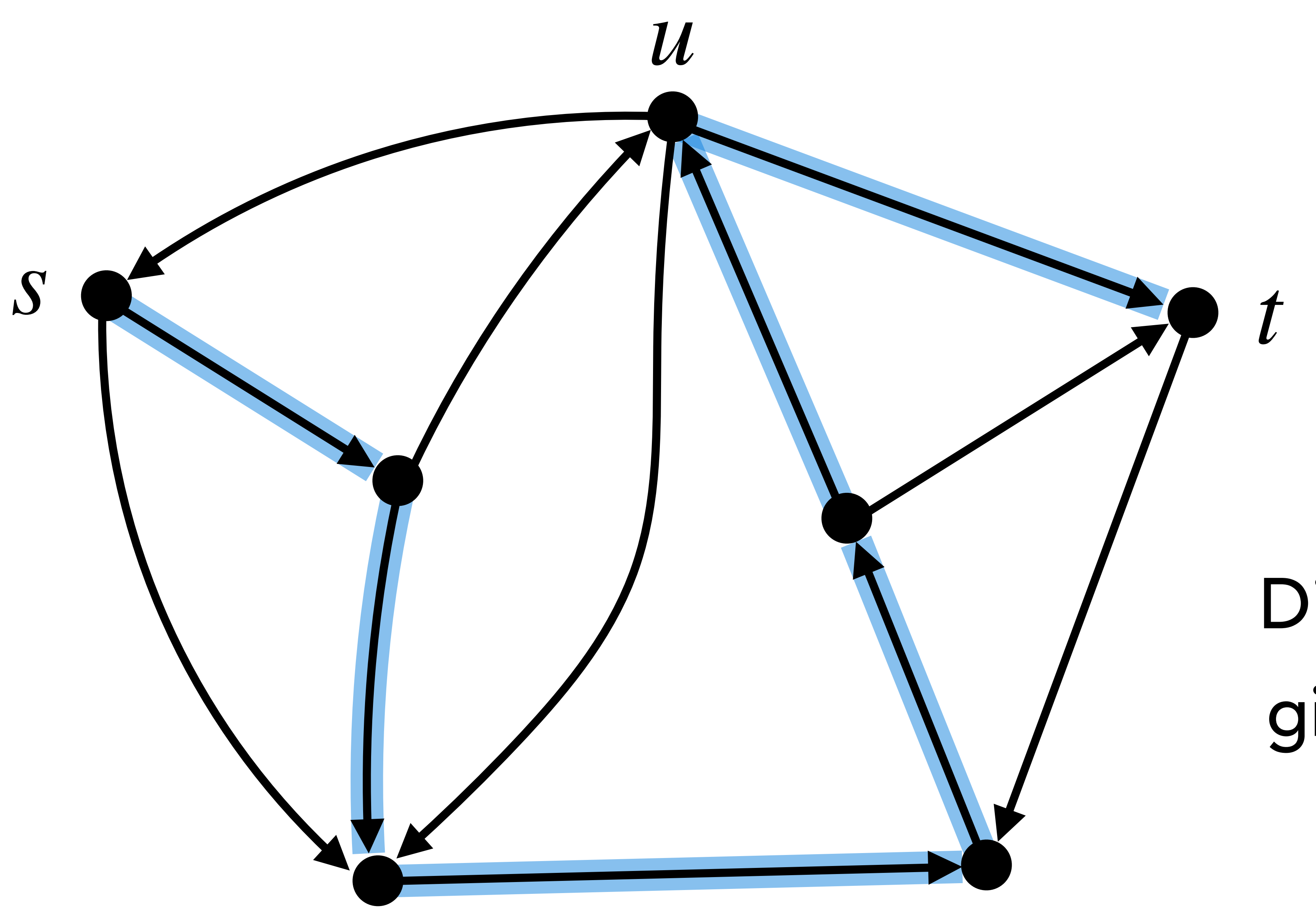


Directed hamiltonian path in G gives a hamiltonian path in G'.

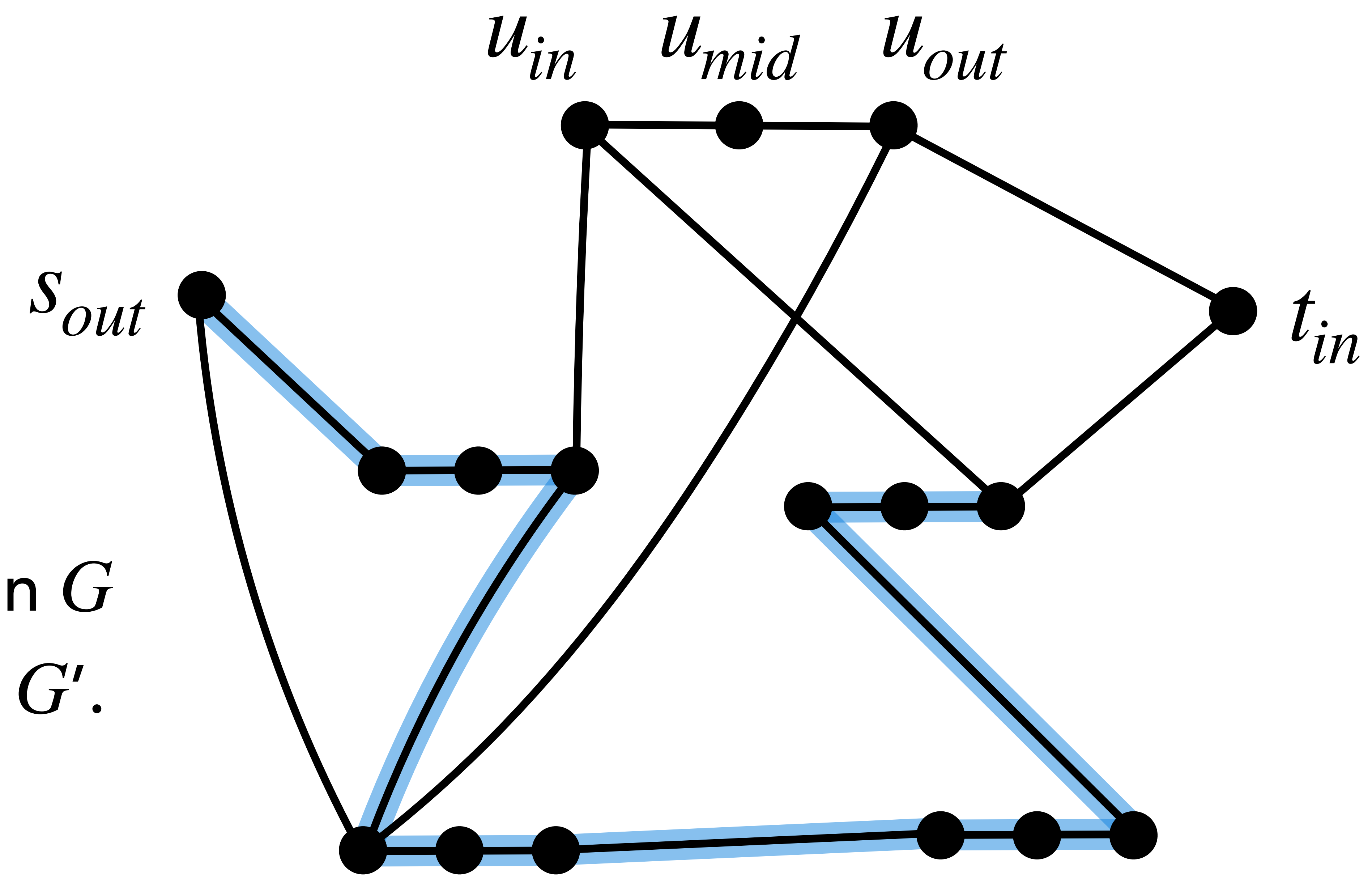


DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$):

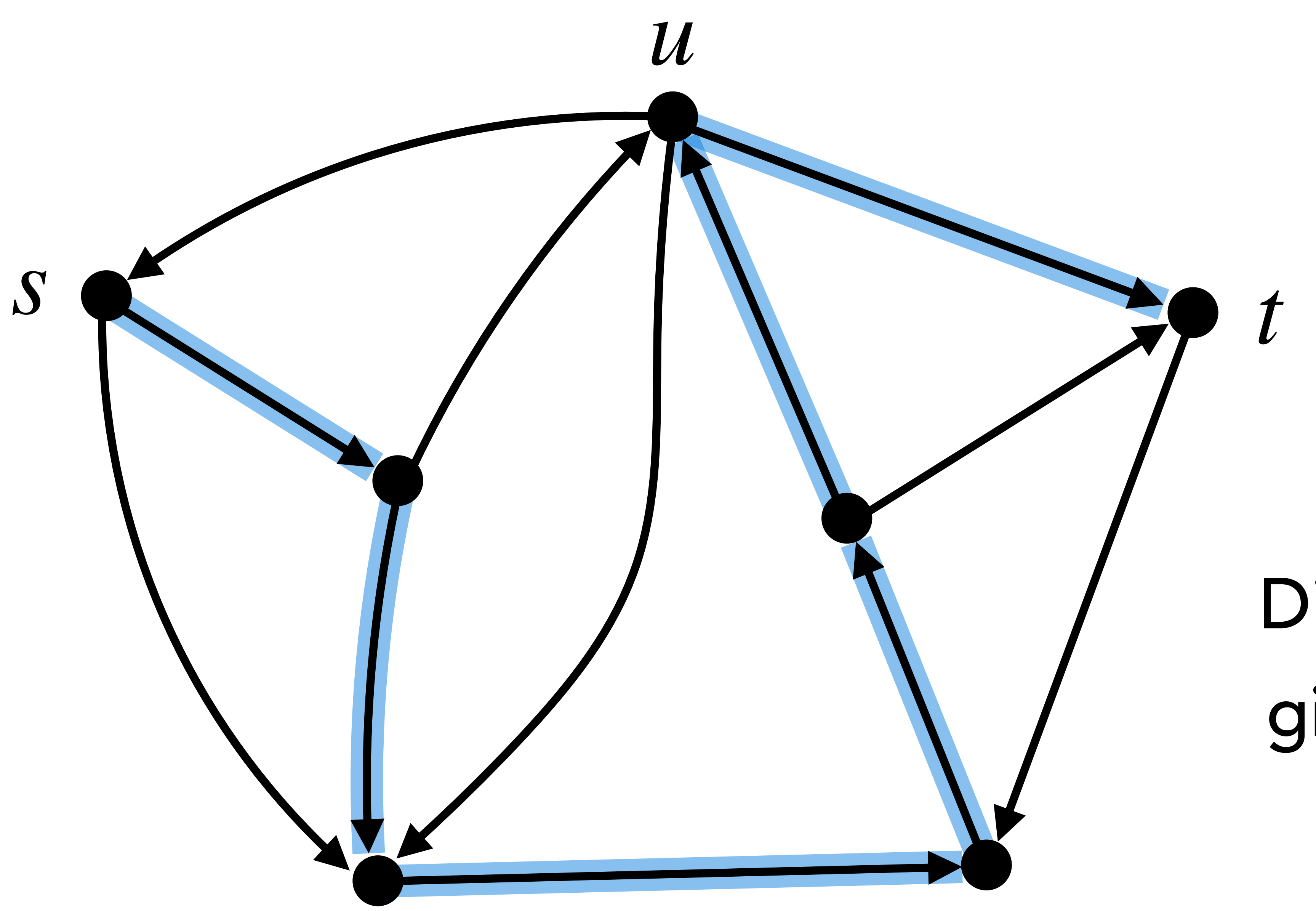


Directed hamiltonian path in G gives a hamiltonian path in G'.

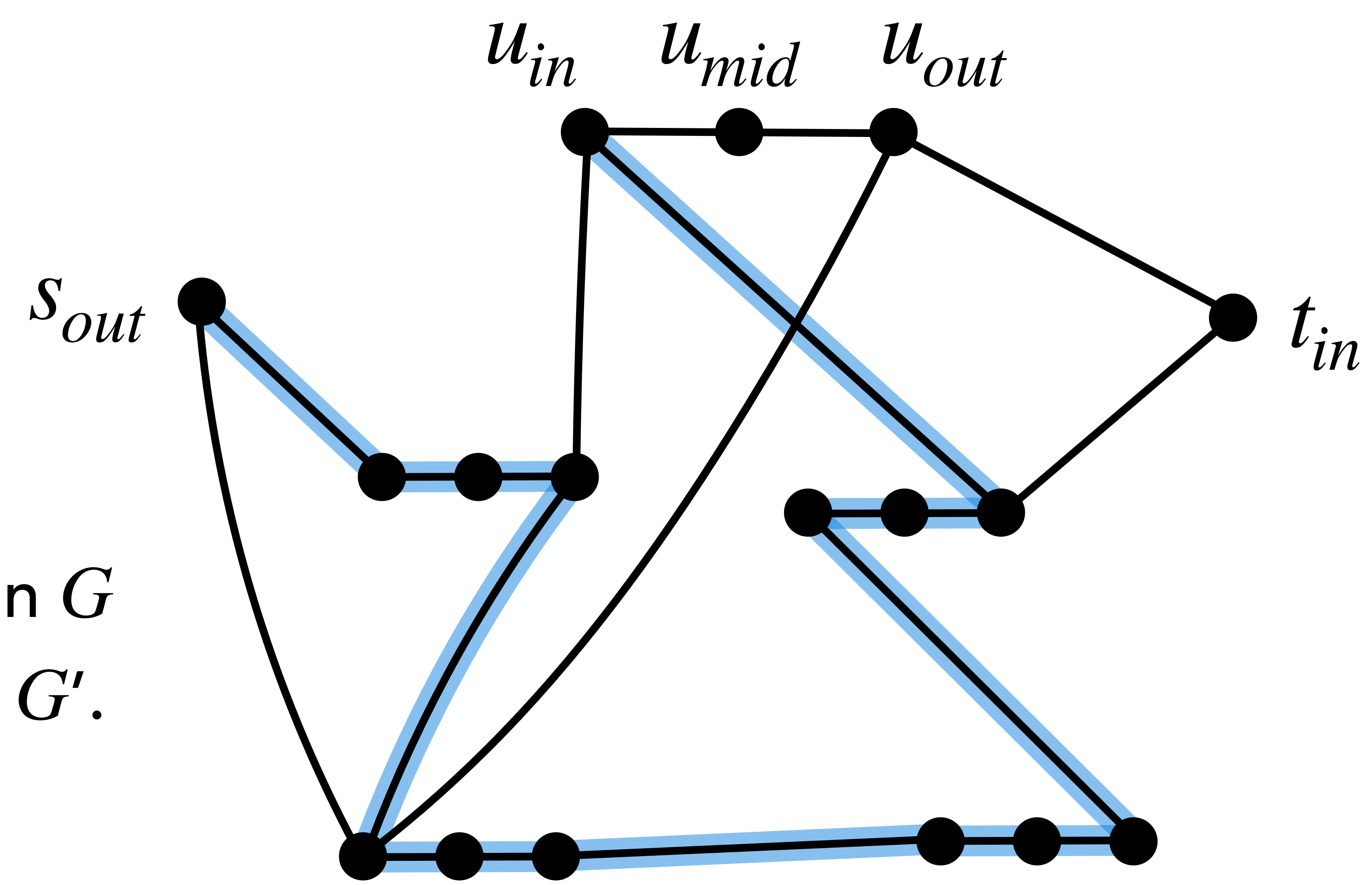


DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longrightarrow$):

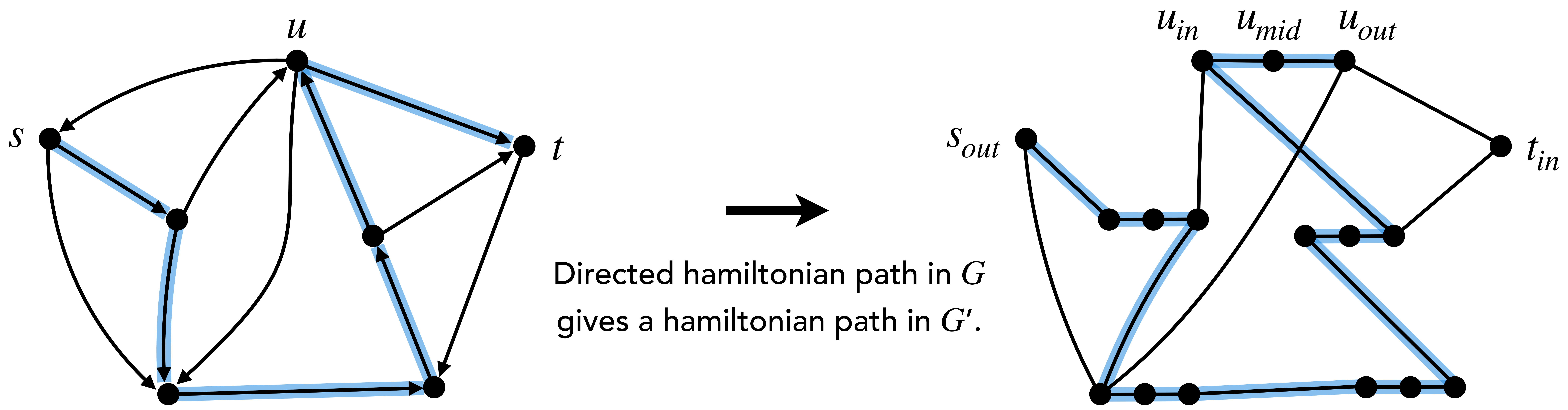


Directed hamiltonian path in G gives a hamiltonian path in G'.



$DirHampath \leq_p Hampath$

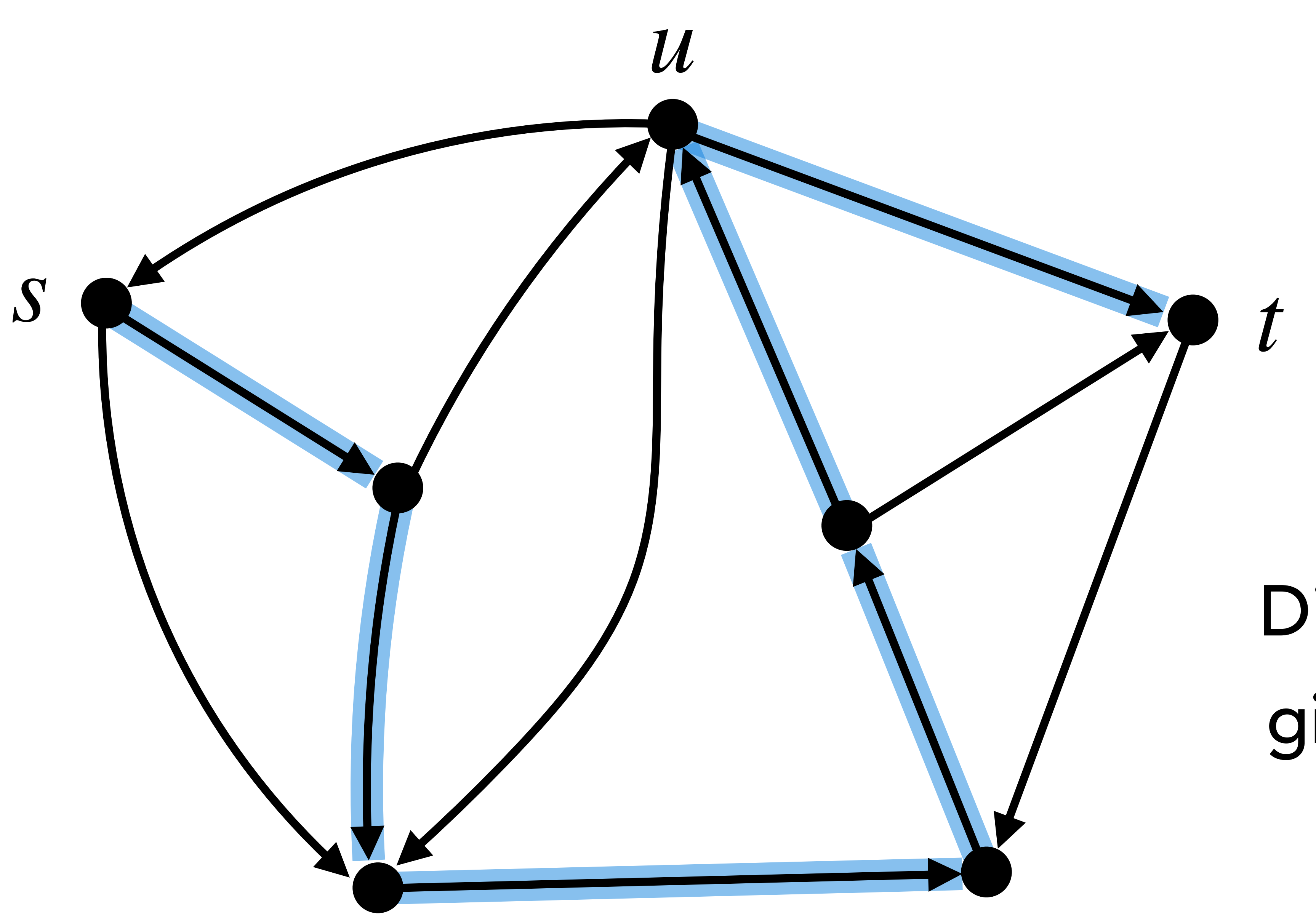
Correctness of reduction ($\Longrightarrow$):



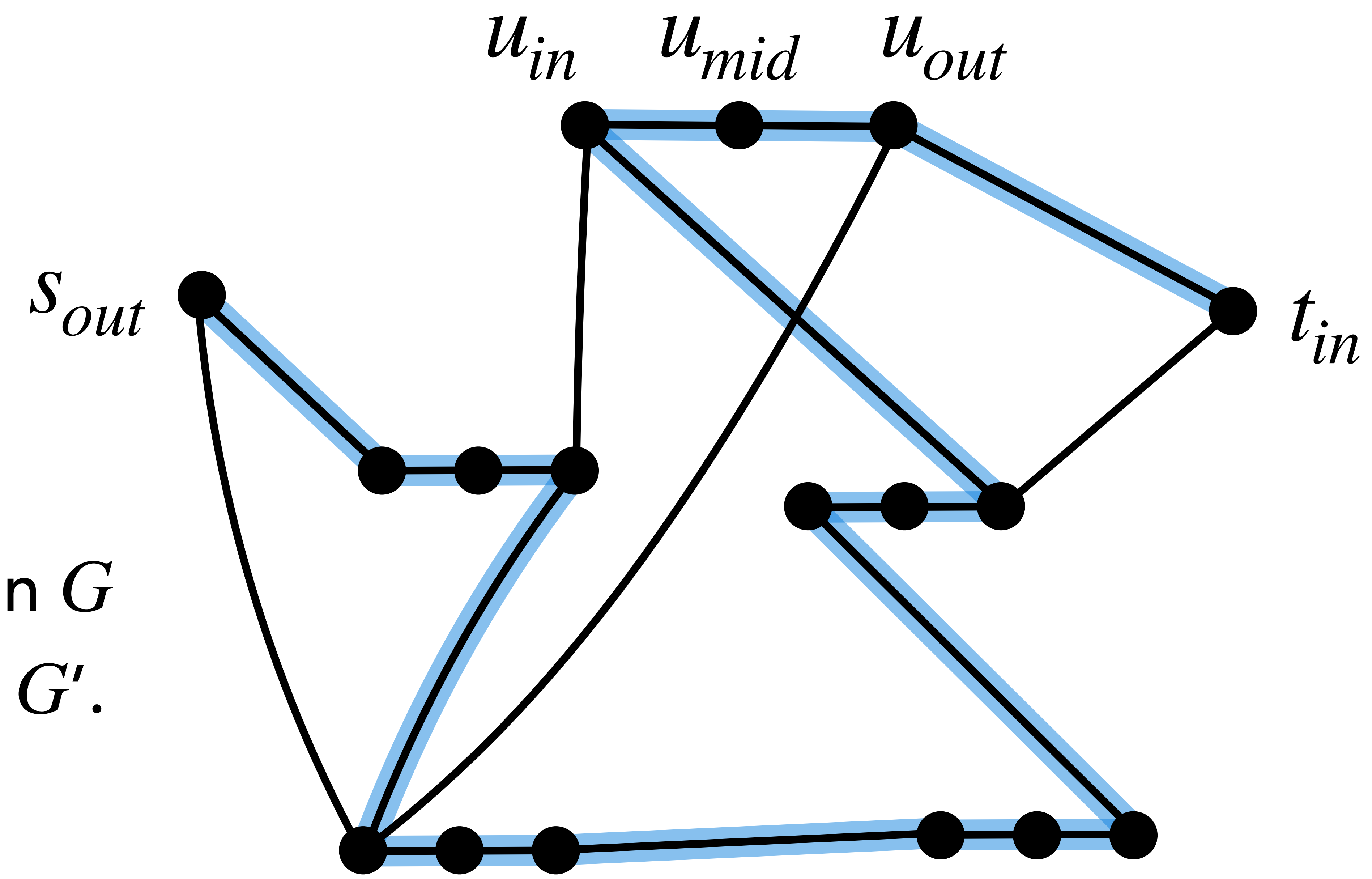
Directed hamiltonian path in G gives a hamiltonian path in G'.

$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longrightarrow$):

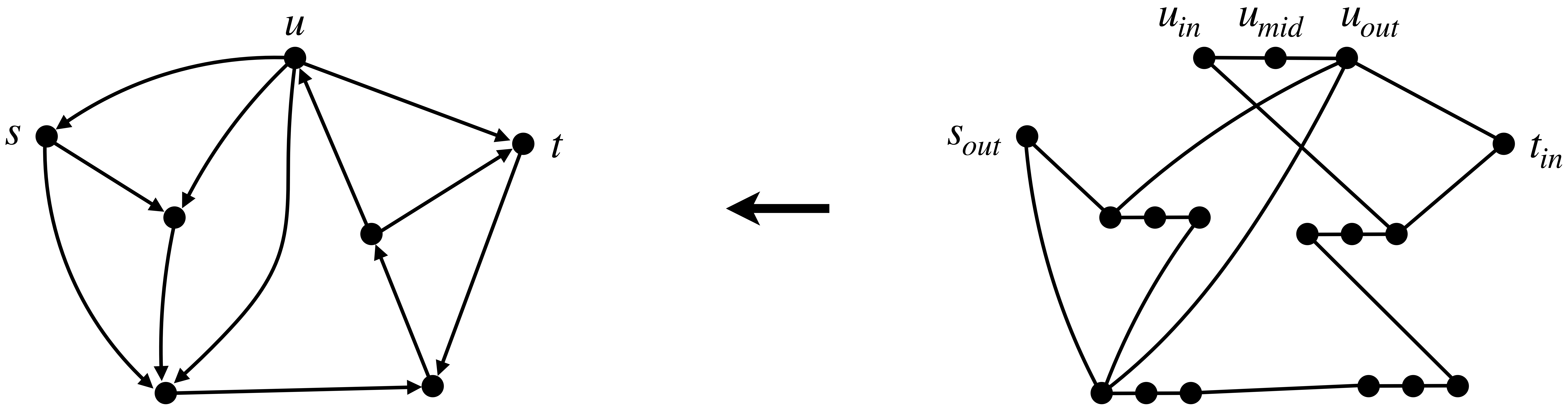


Directed hamiltonian path in G gives a hamiltonian path in G'.



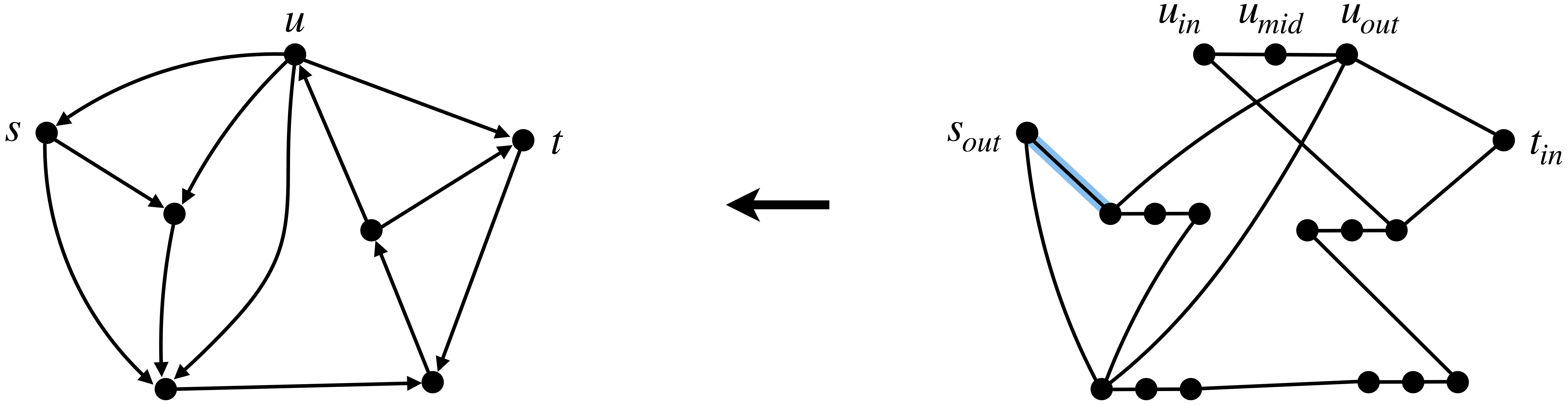
$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longleftarrow$):



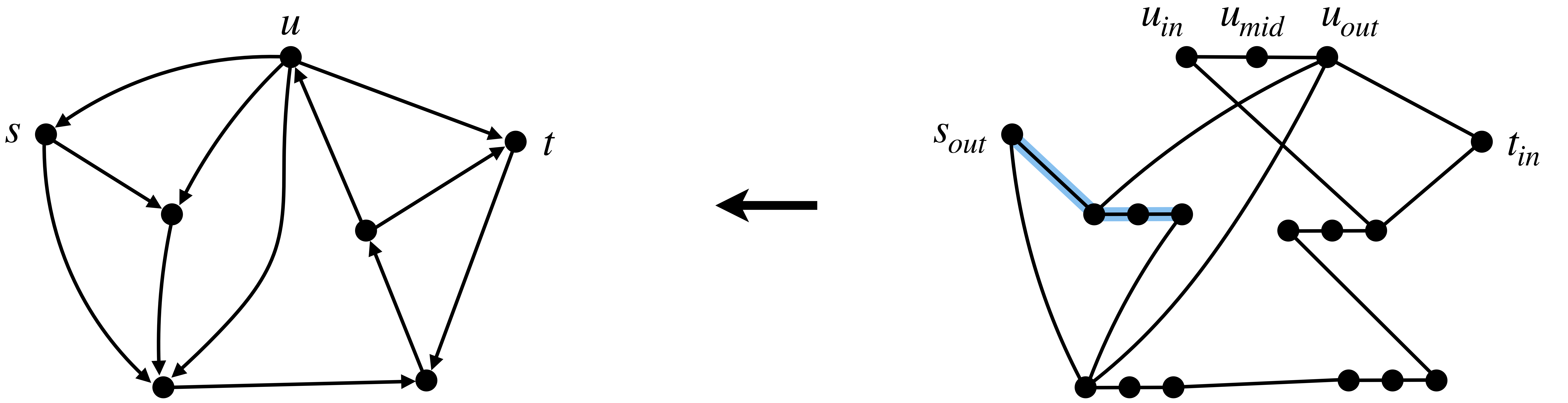
DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longleftarrow)$**:**



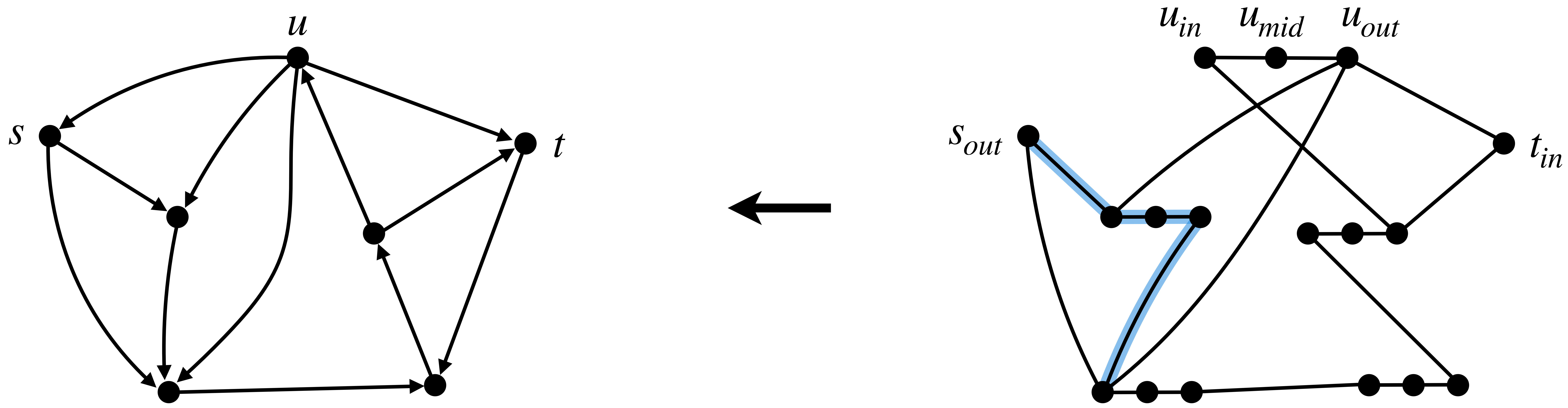
$DirHampath \leq_p Hampath$

Correctness of reduction $(\Longleftarrow)$**:**



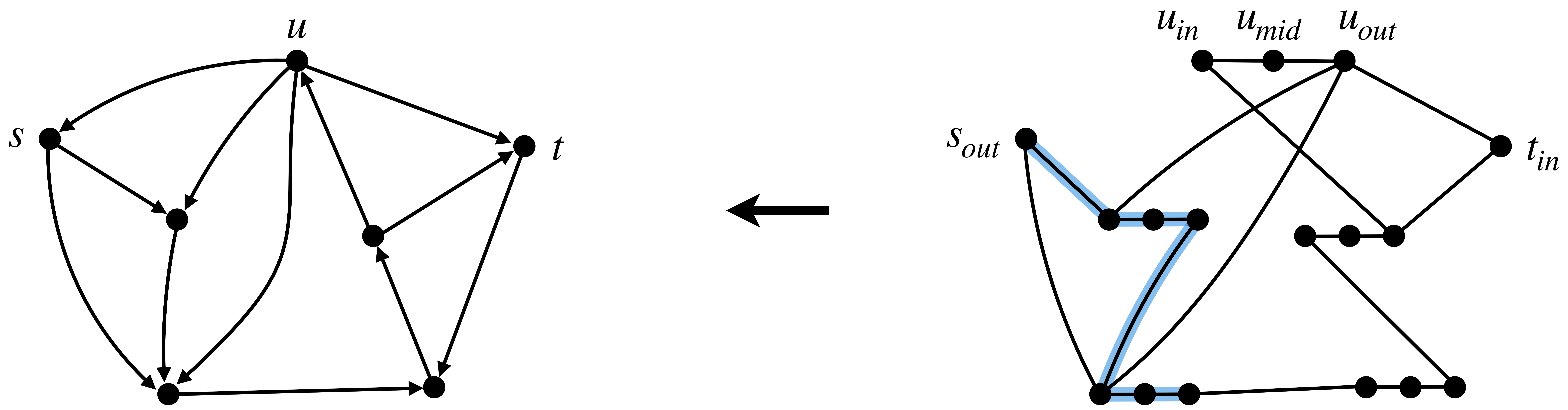
DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longleftarrow)$**:**



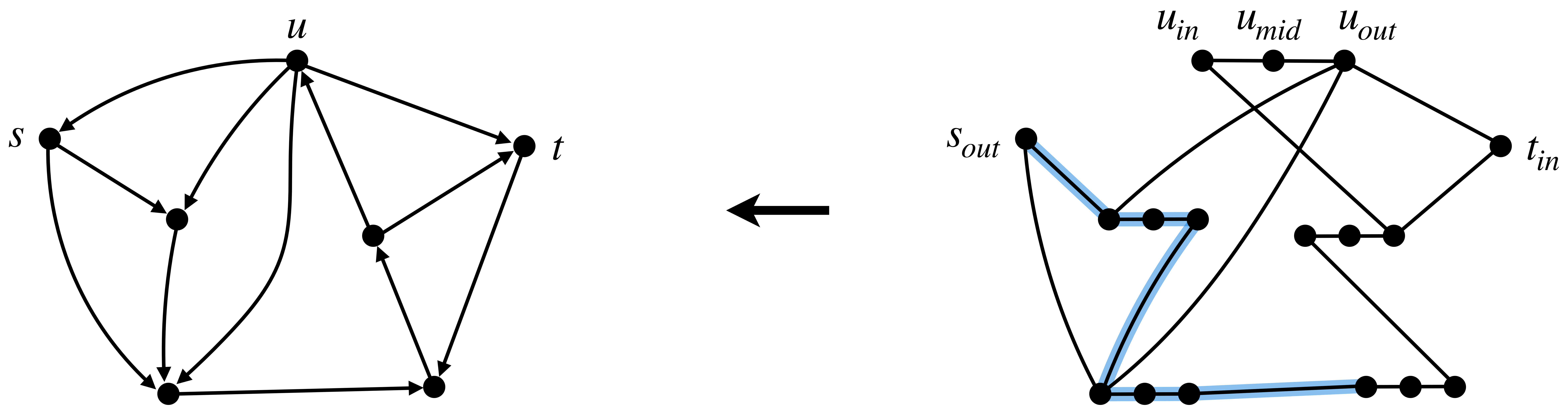
DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longleftarrow)$**:**



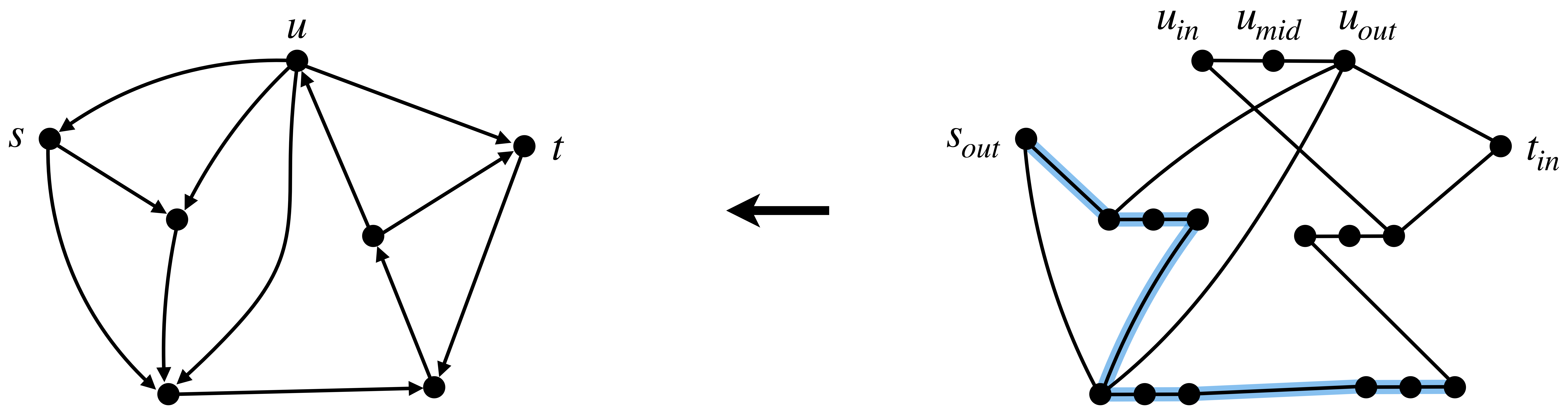
$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longleftarrow$)**:**



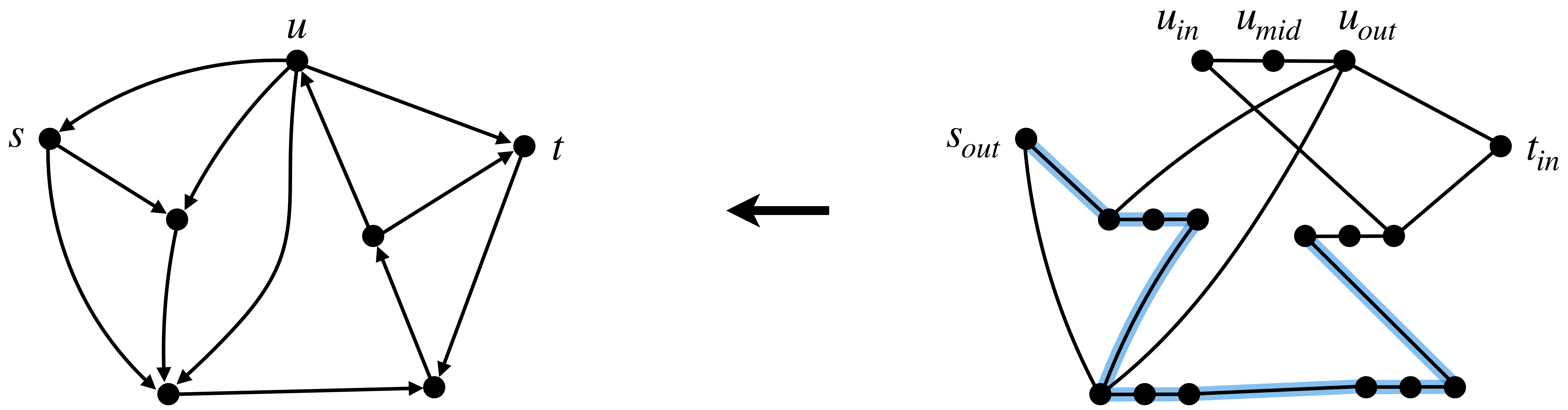
DirHampath $\leq_p$ *Hampath*

Correctness of reduction ($\Longleftarrow$):



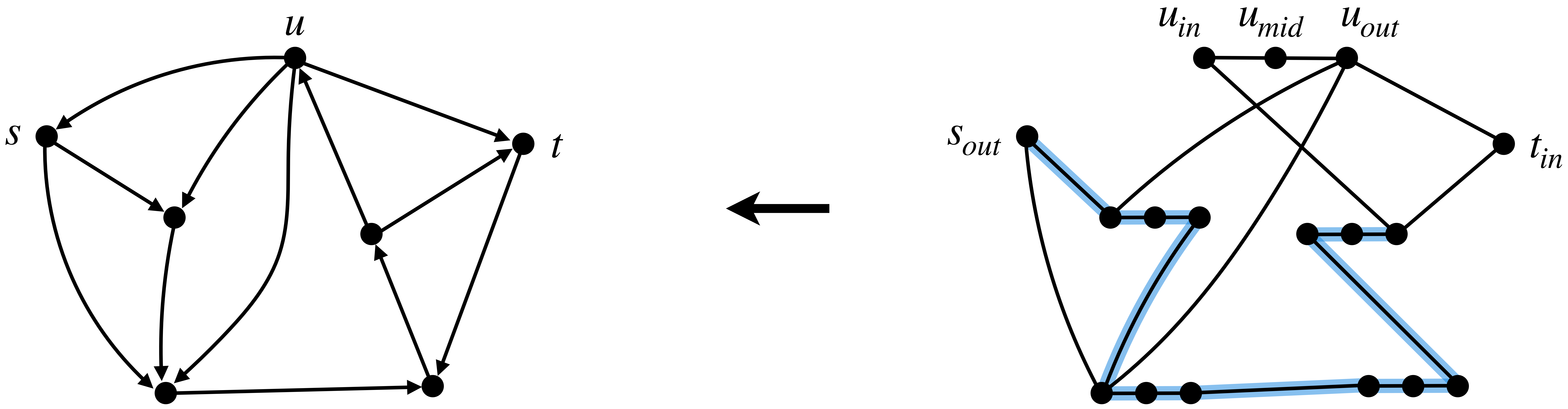
$DirHampath \leq_p Hampath$

Correctness of reduction $(\Longleftarrow)$**:**



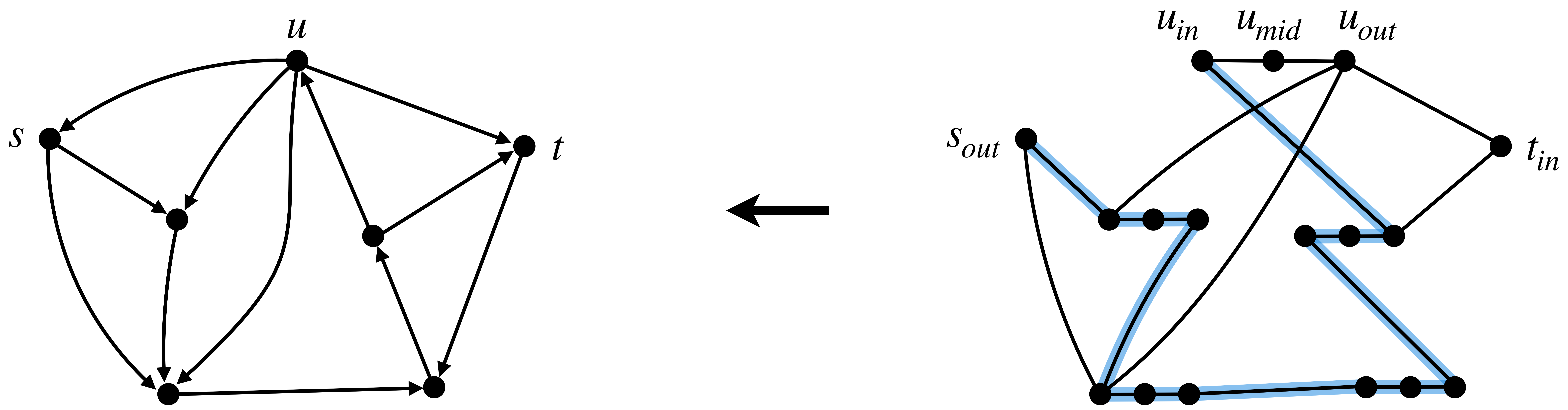
$DirHampath \leq_p Hampath$

Correctness of reduction ($\Longleftarrow$):



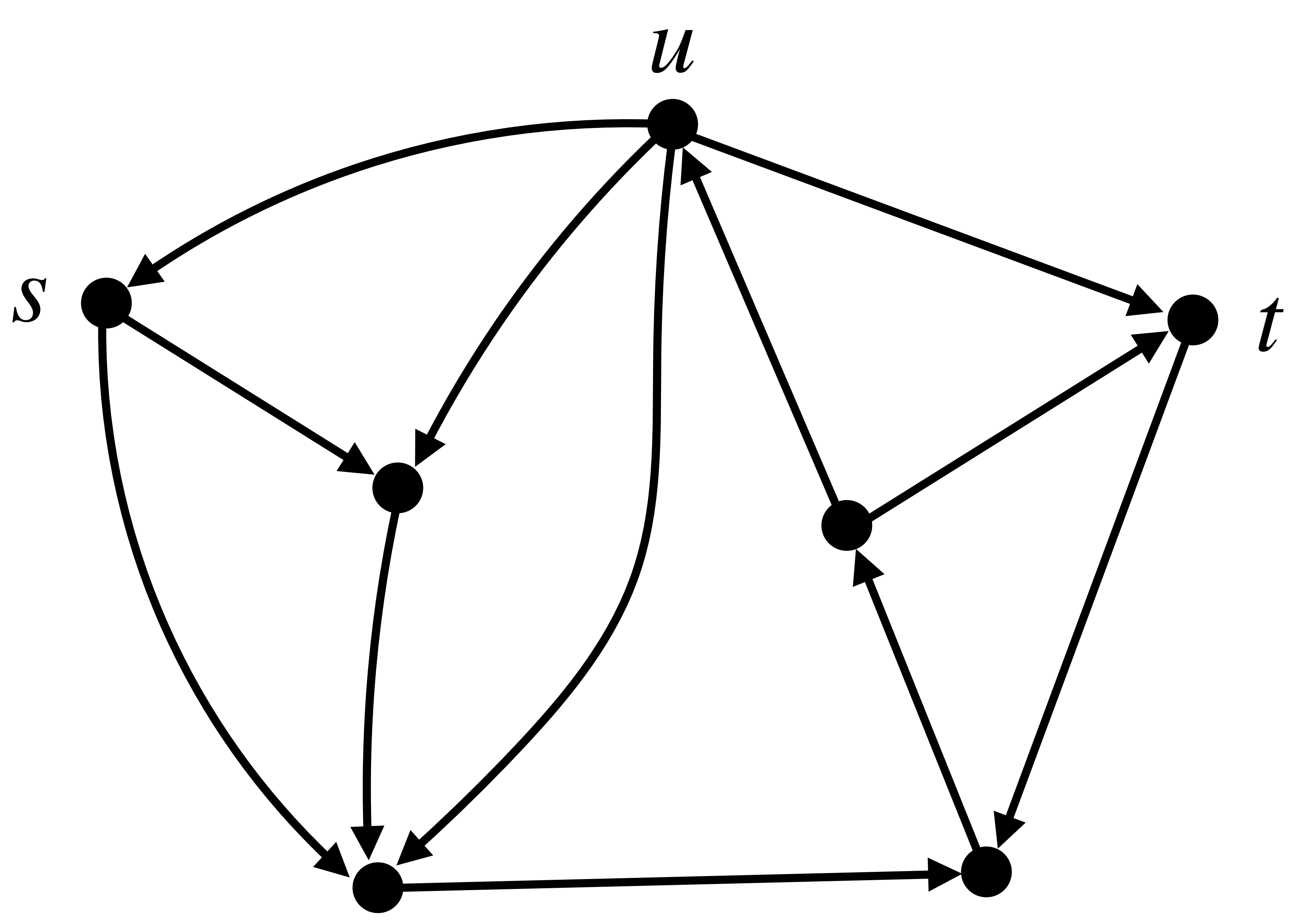
$DirHampath \leq_p Hampath$

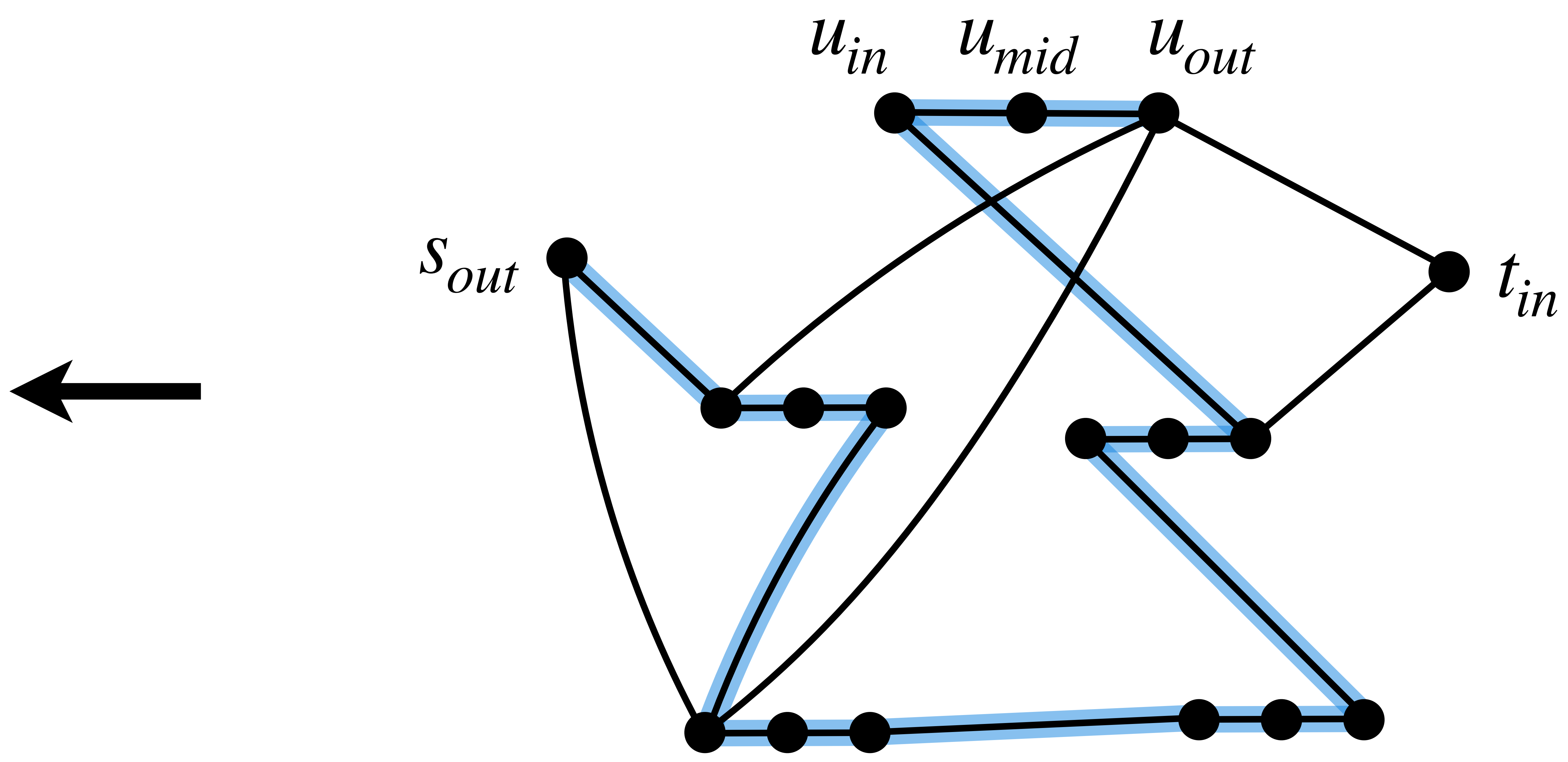
Correctness of reduction ($\Longleftarrow$):



DirHampath $\leq_p$ *Hampath*

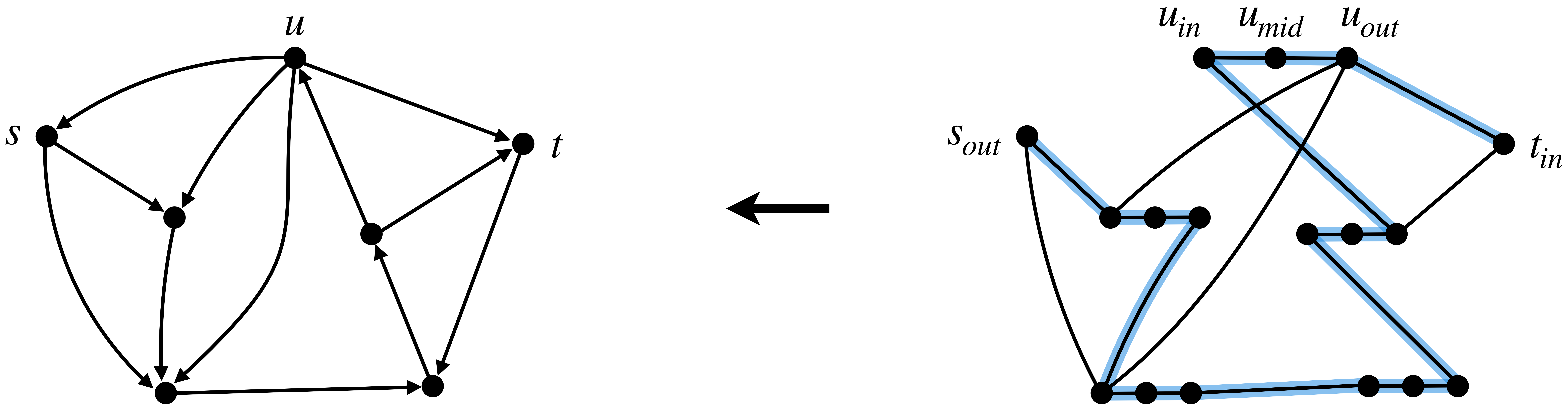
Correctness of reduction $(\Longleftarrow)$**:**





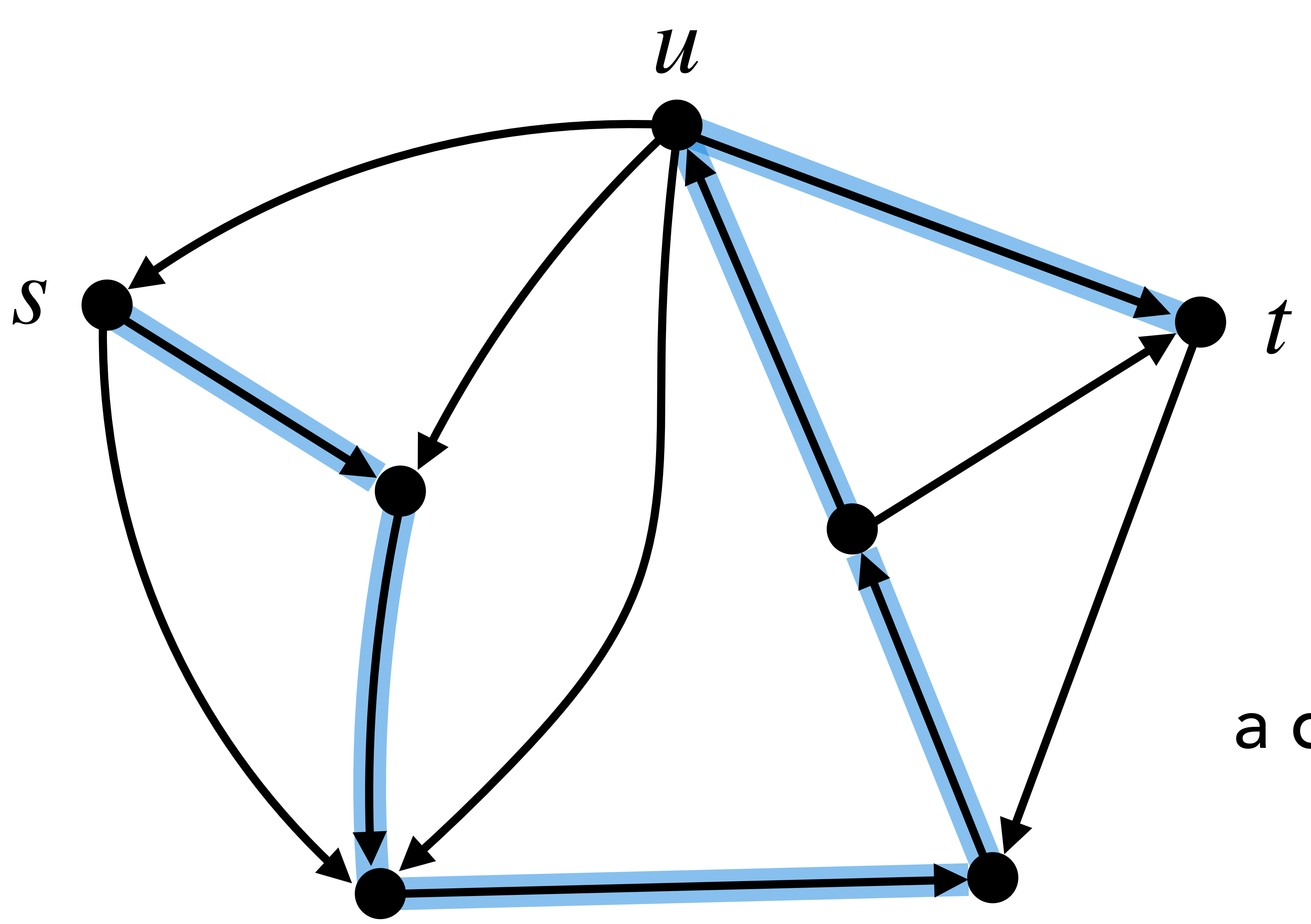
DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longleftarrow)$**:**

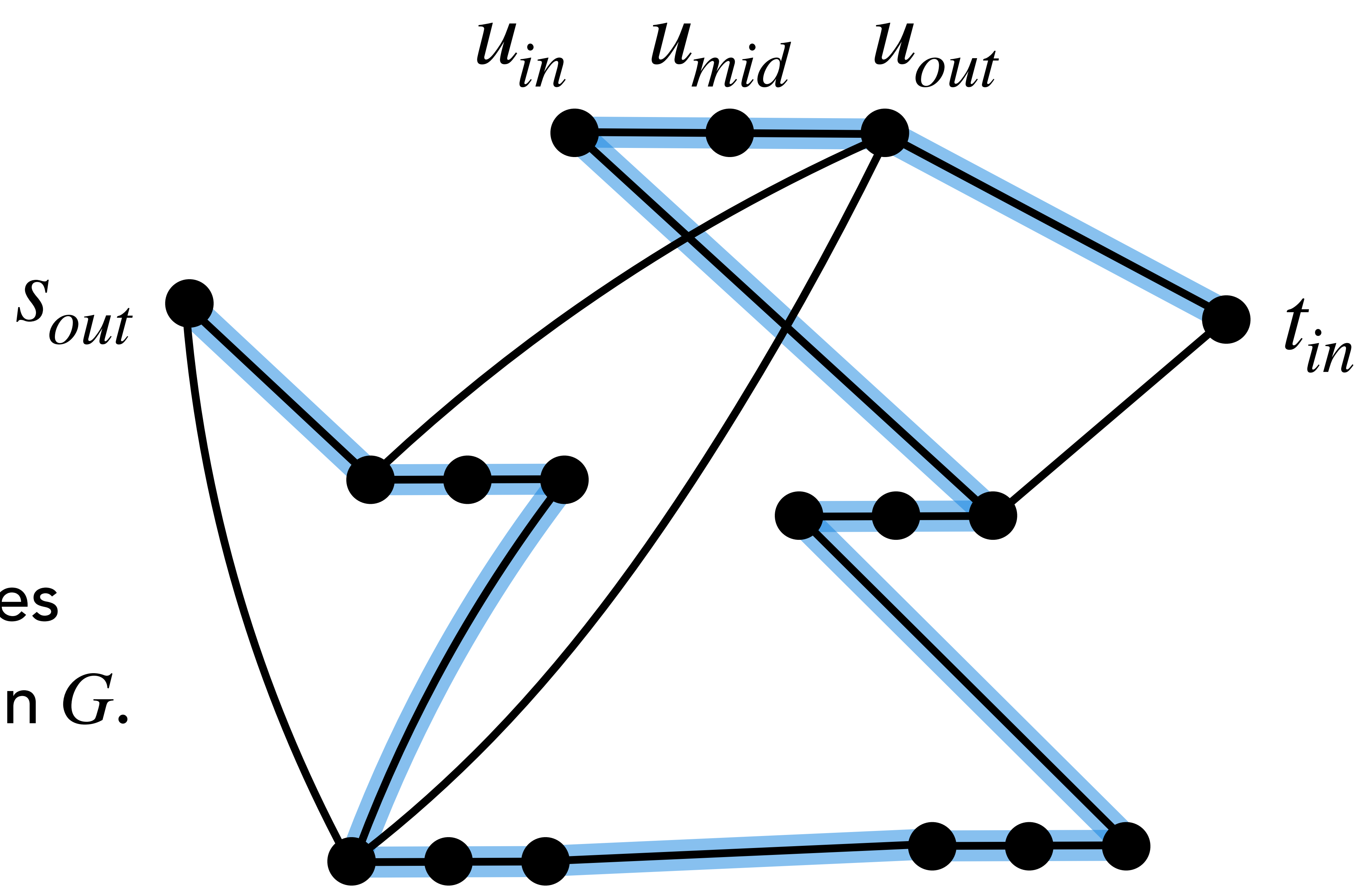


DirHampath $\leq_p$ *Hampath*

Correctness of reduction $(\Longleftarrow)$**:**



Hamiltonian path in G' gives
a directed hamiltonian path in G.



DirHampath $\leq_p$ Hampath

$\langle G, s, t\rangle \rightarrow \langle G', s', t'\rangle$:

- Vertices of G':
  - For s have s_{out}, t have t_{in}.
  - For any other vertex u have a 3 vertices u_{in}, u_{mid}, u_{out}.
- Edges of G':
  - Ignore the incoming edges to s and outgoing edges from t.
  - For every edge (u, v) have an edge $\{u_{out}, v_{in}\}$.
  - Edges $\{u_{in}, u_{mid}\}$ and $\{u_{mid}, u_{out}\}$.
- $s' = s_{out}$, $t' = t_{in}$.

← Prove the correctness formally yourself.

Hampath $\leq_p$ *Hamcycle*

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

Hampath $\leq_p$ *Hamcycle*

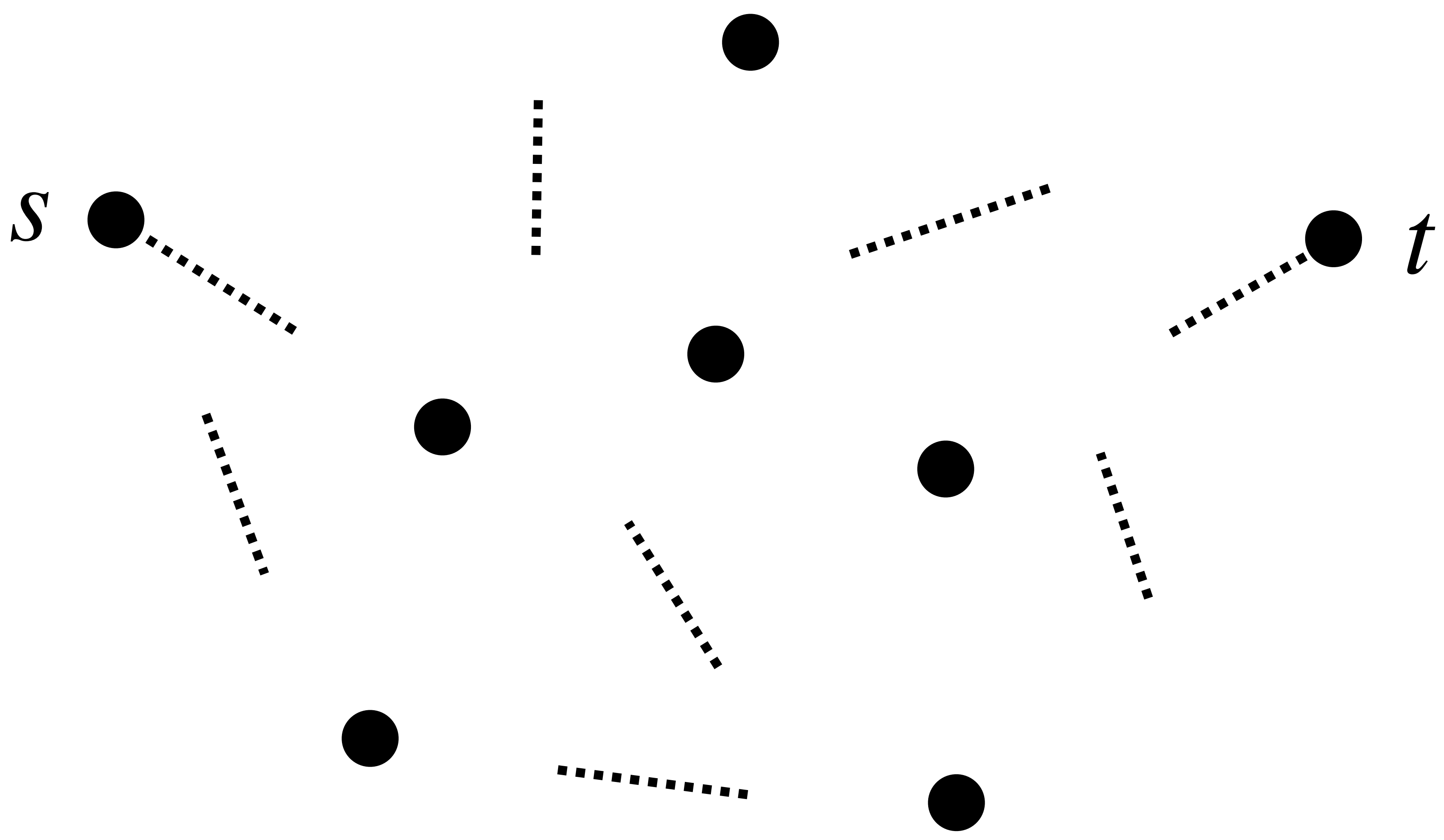
- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G' \rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \to \langle G' \rangle$:

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

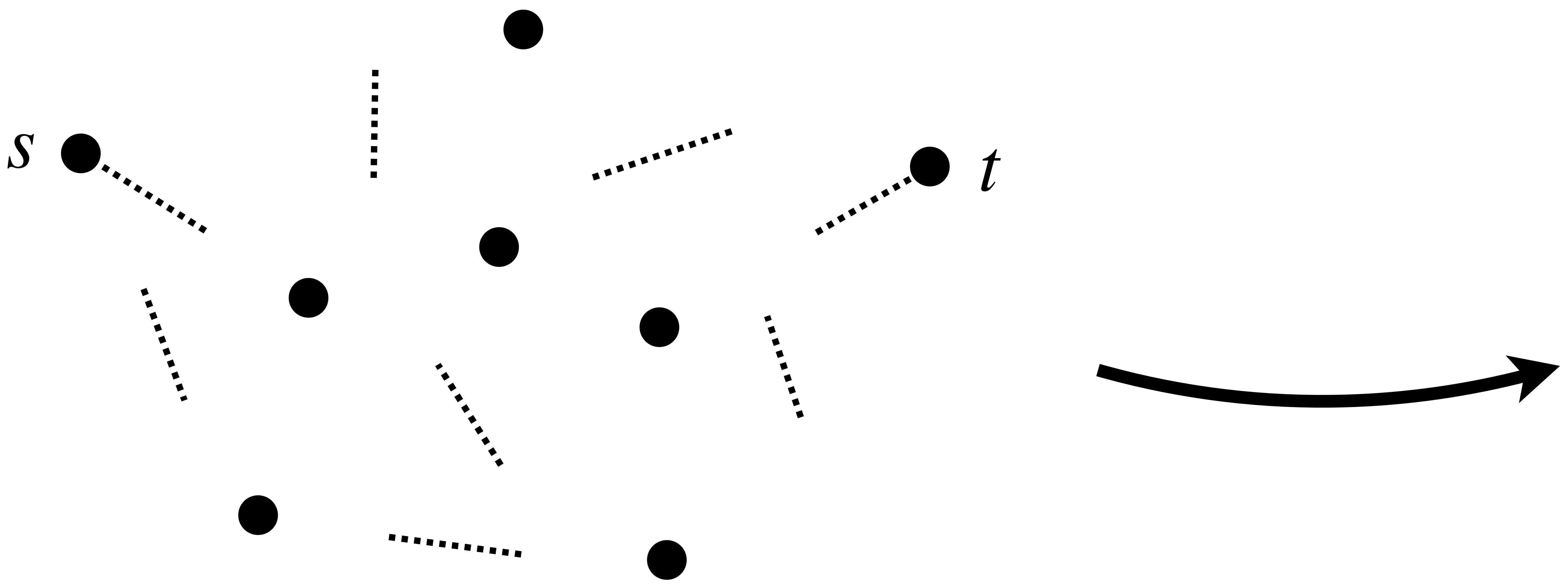
$\langle G, s, t\rangle \to \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

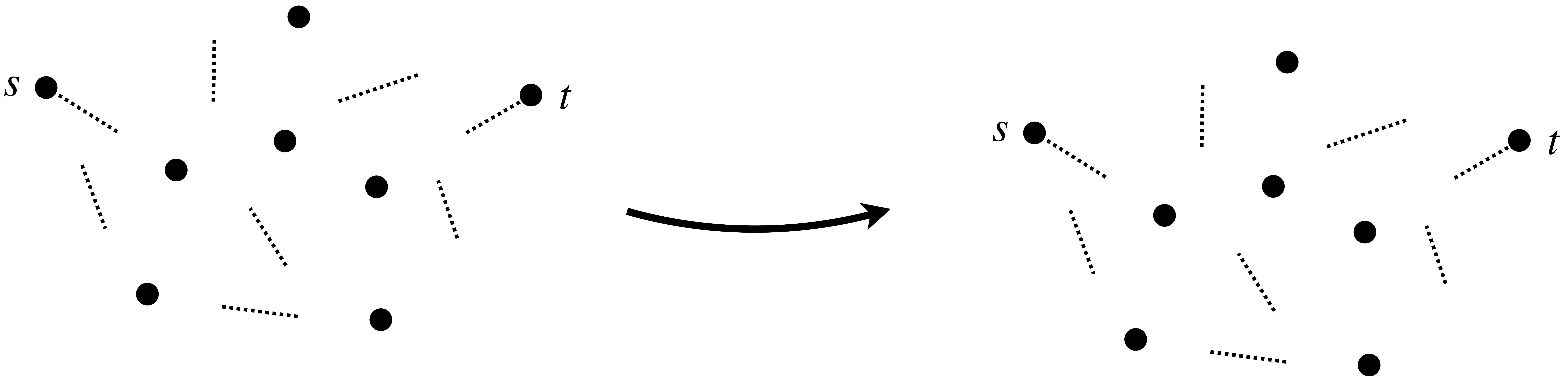
$\langle G, s, t\rangle \to \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

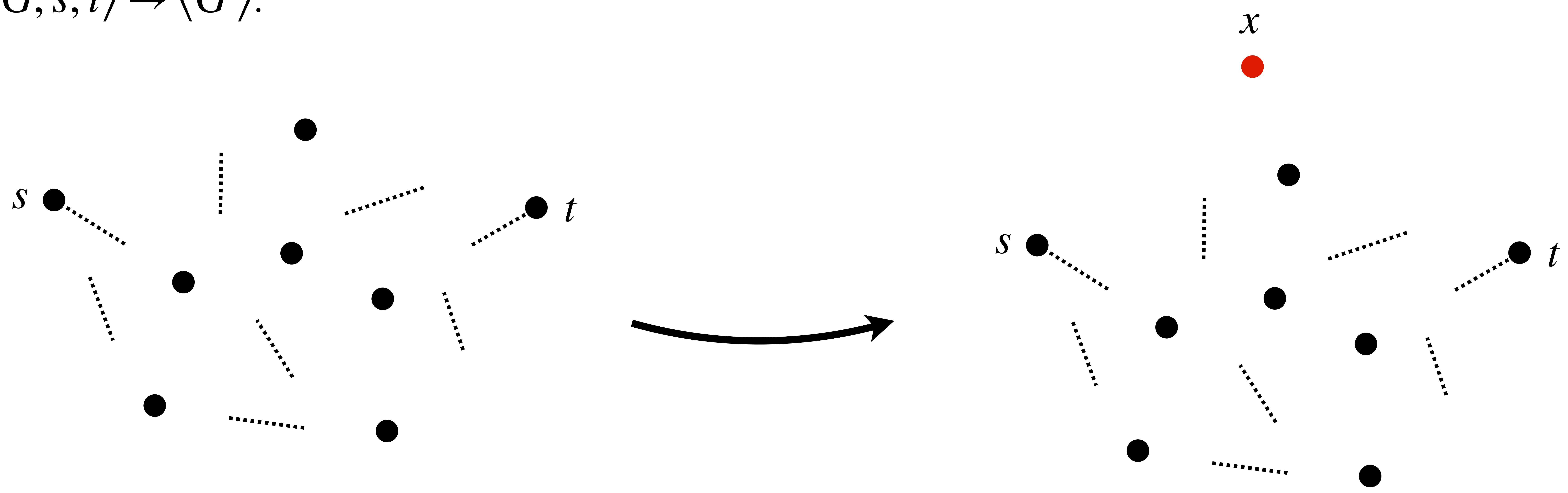
$\langle G, s, t\rangle \to \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

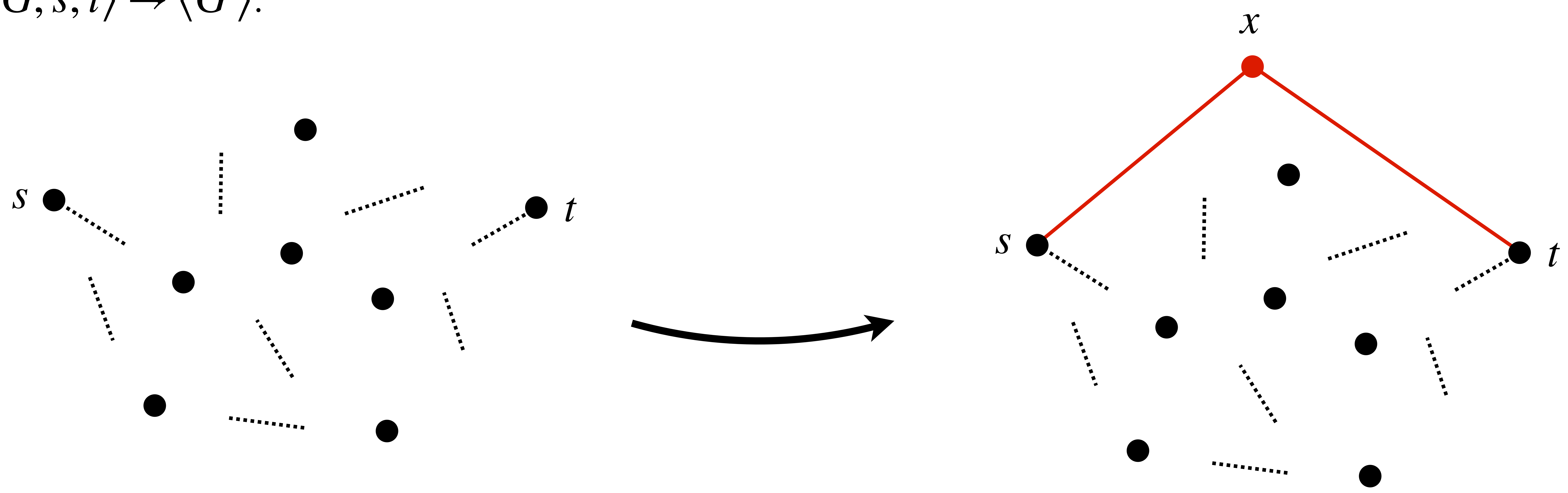
$\langle G, s, t\rangle \to \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

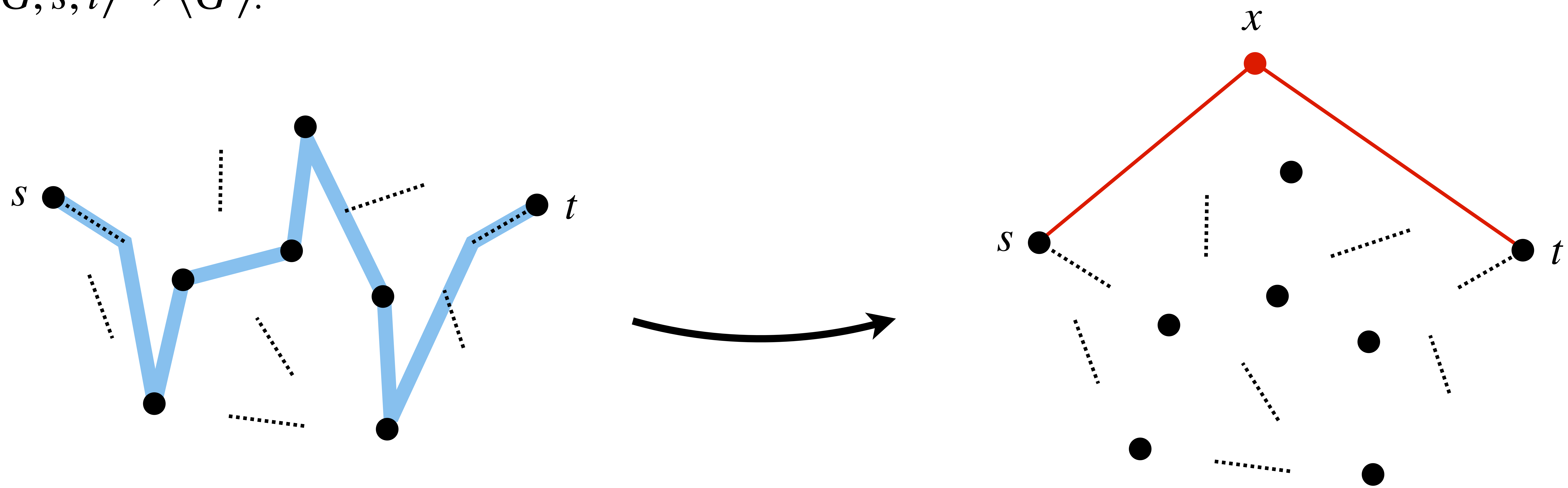
$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

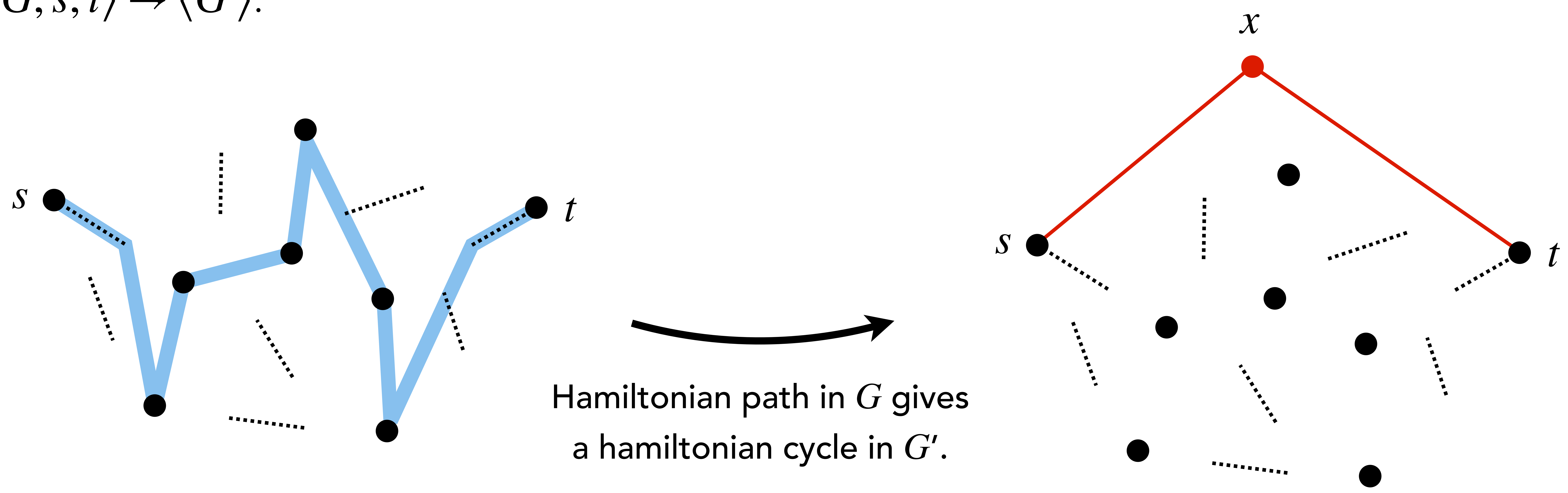
$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

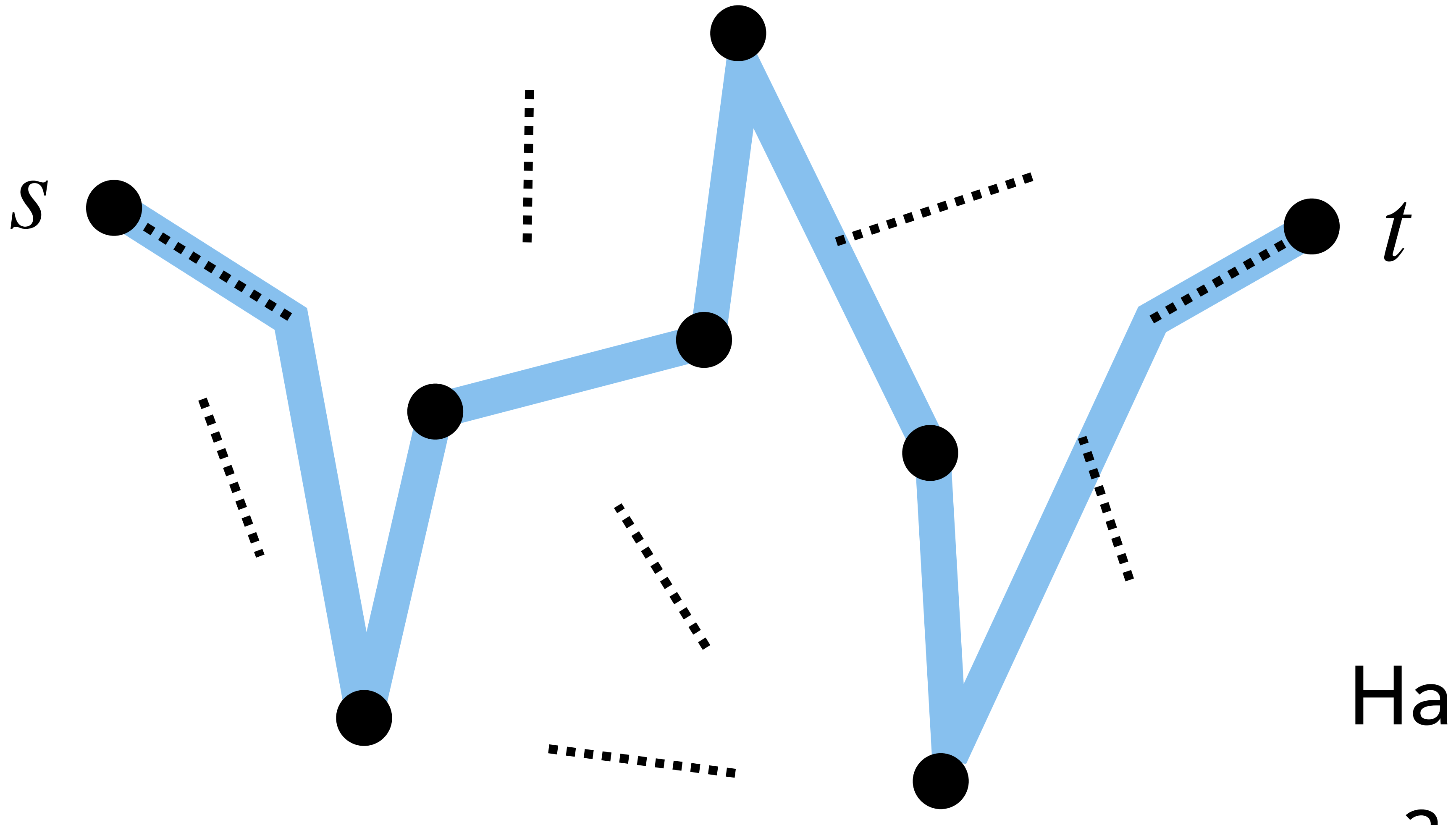
$\langle G, s, t\rangle \to \langle G'\rangle$:

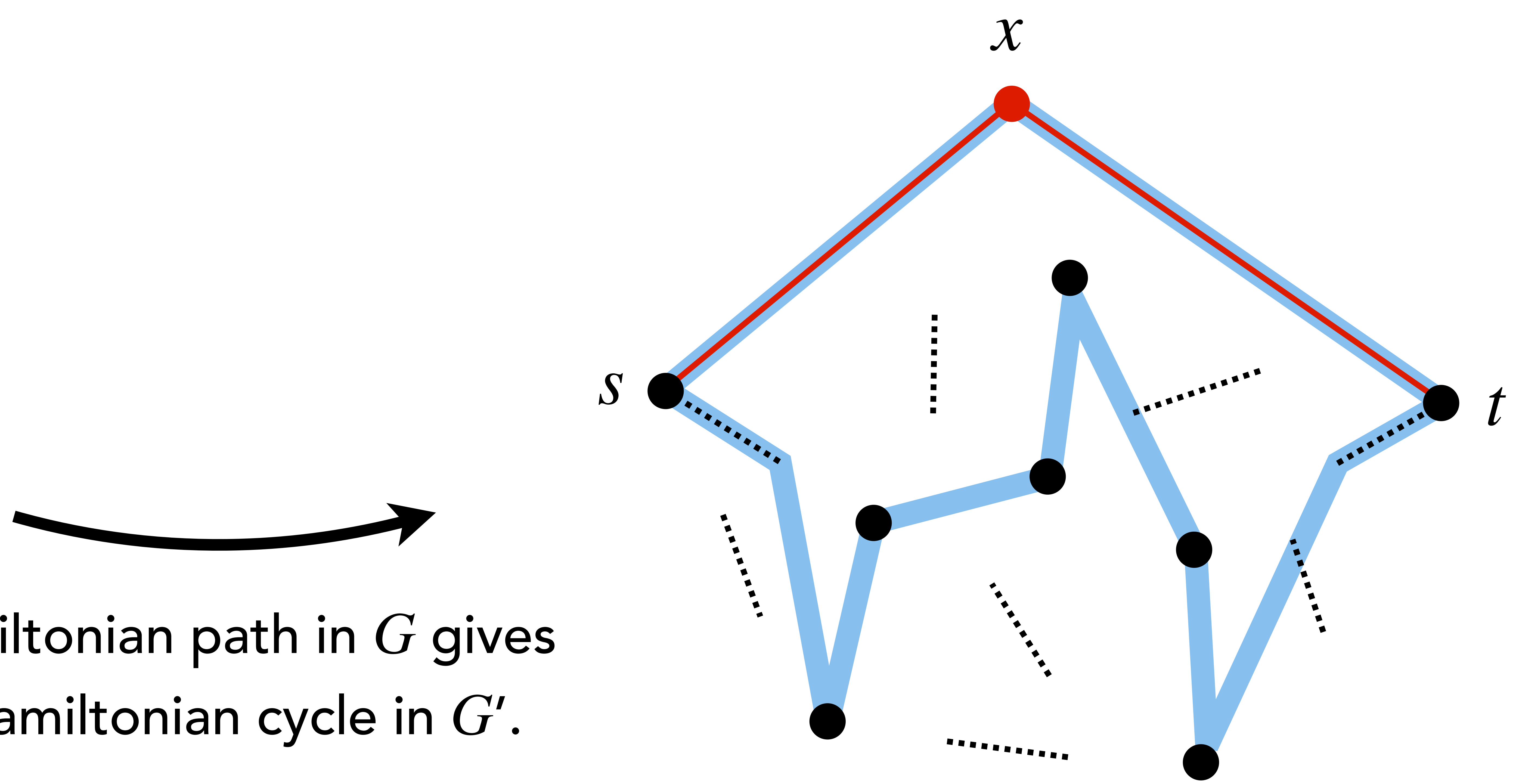


Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:



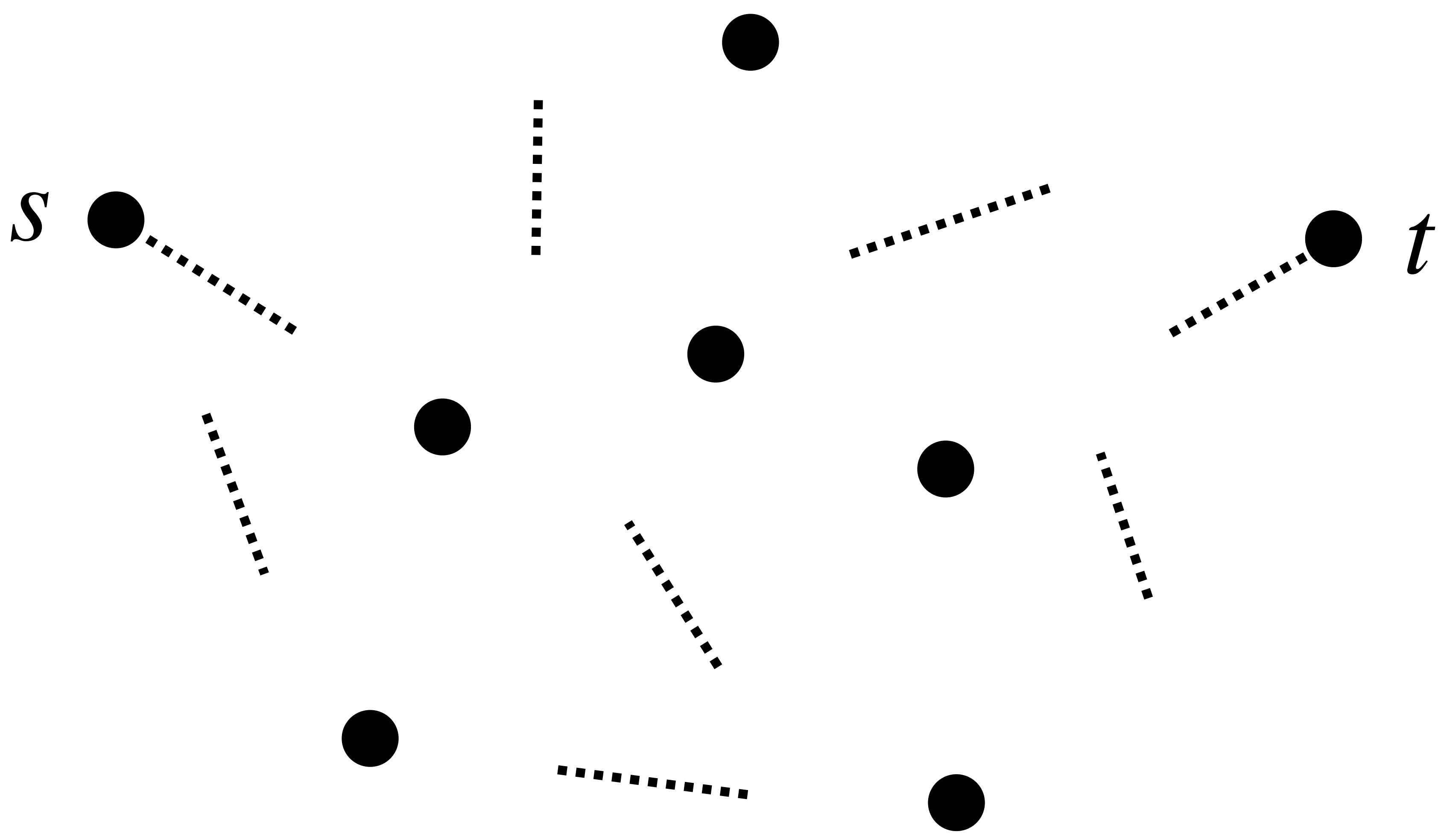


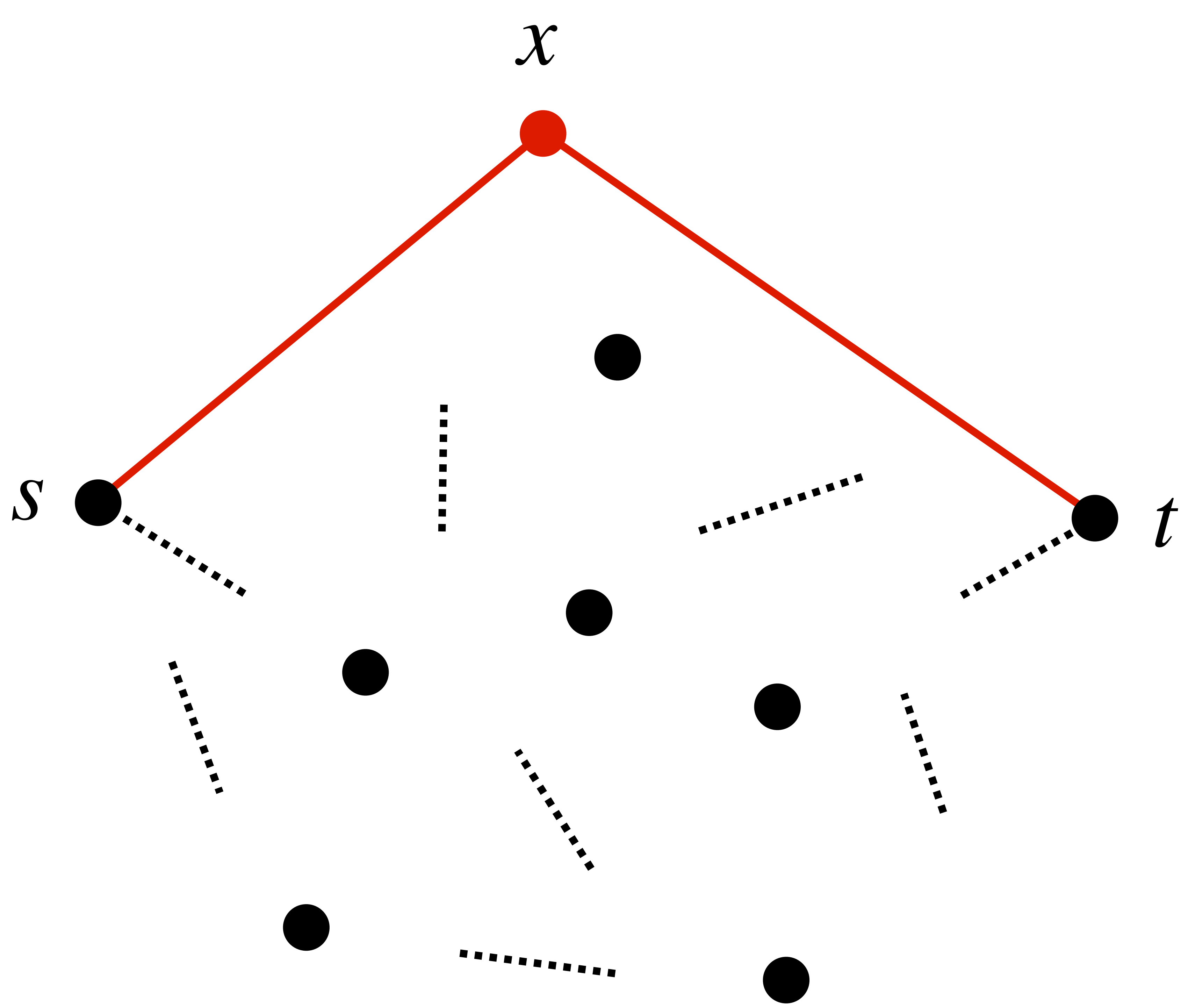
Hamiltonian path in G gives
a hamiltonian cycle in G'.

Hampath $\leq_p$ *Hamcycle*

- *Hampath* = $\{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* = $\{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \to \langle G'\rangle$:

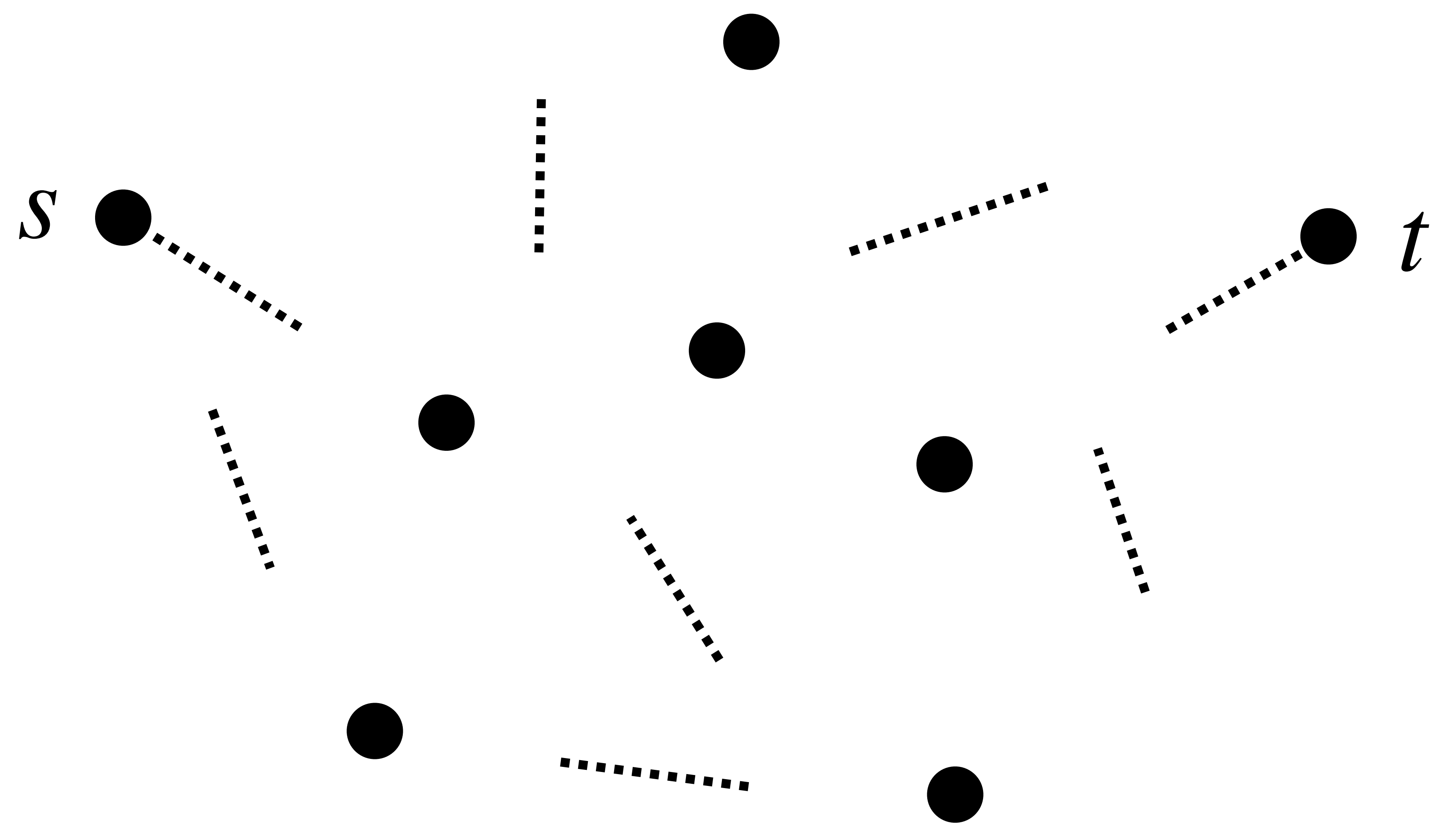


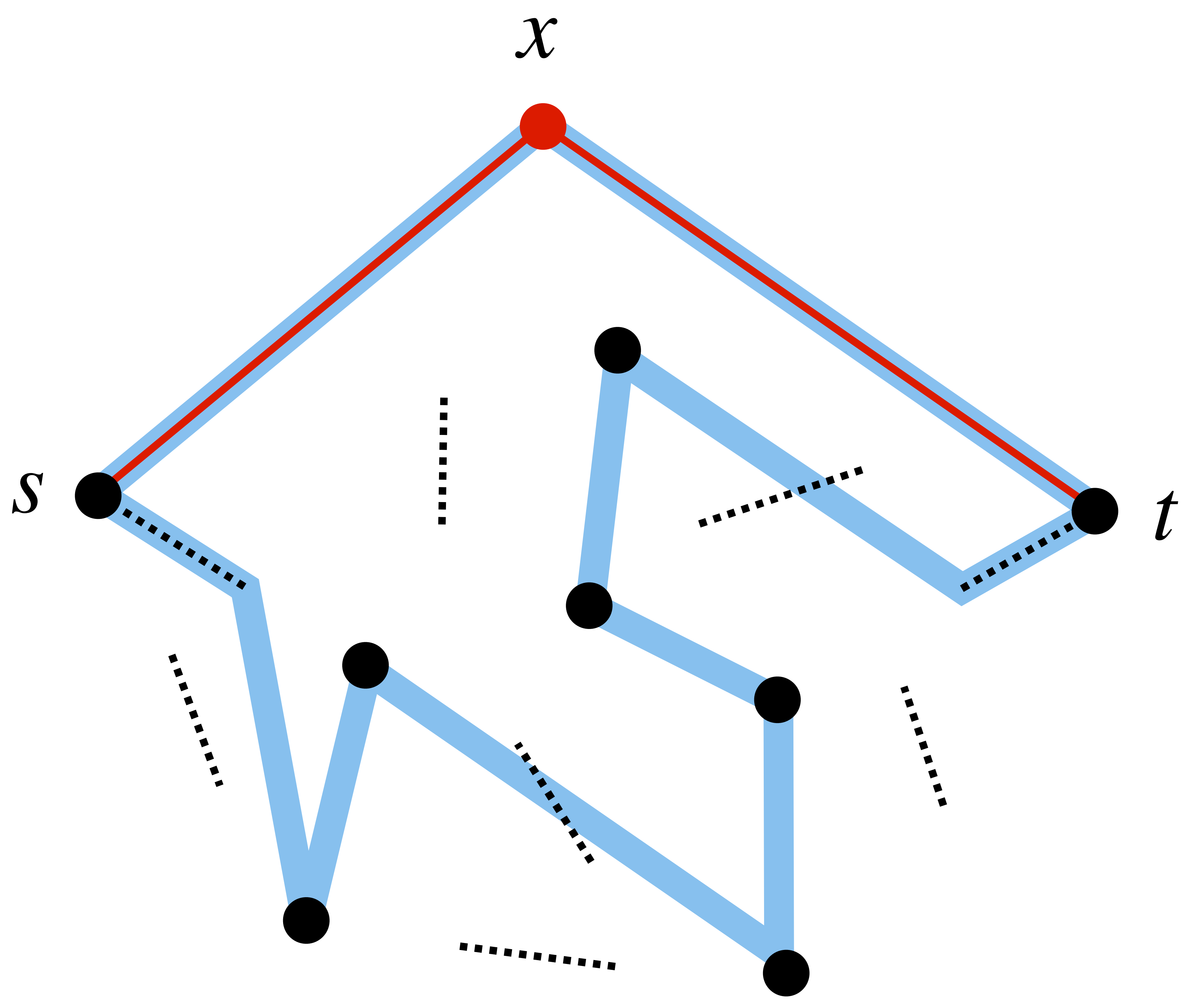


Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \to \langle G'\rangle$:

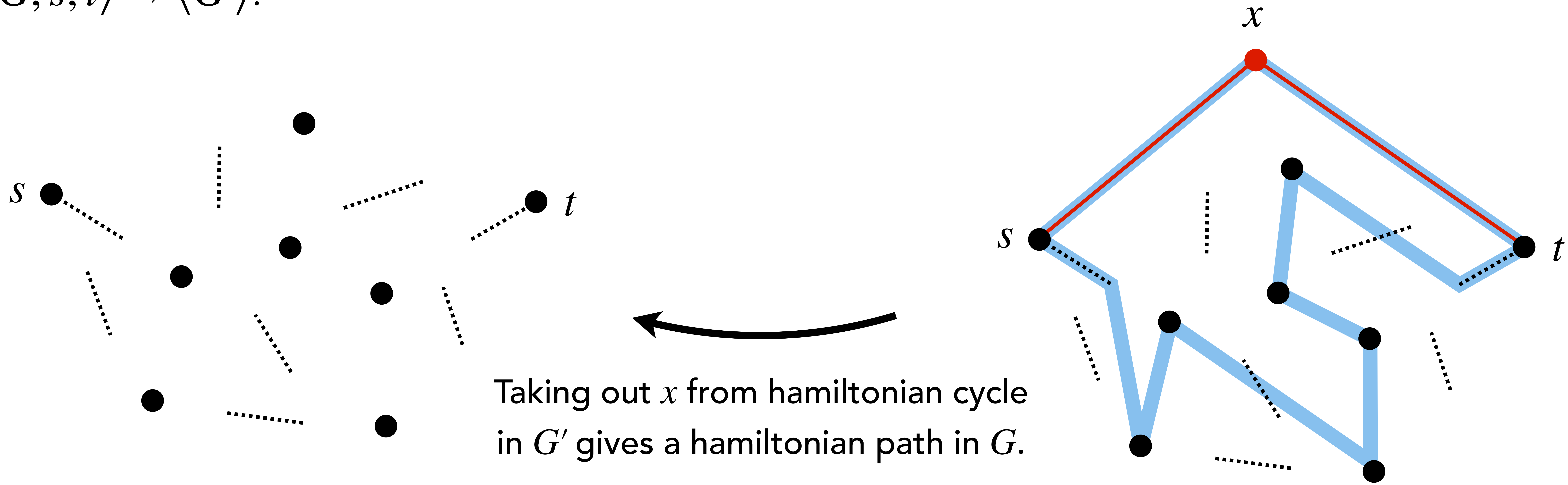




Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t \rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G' \rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

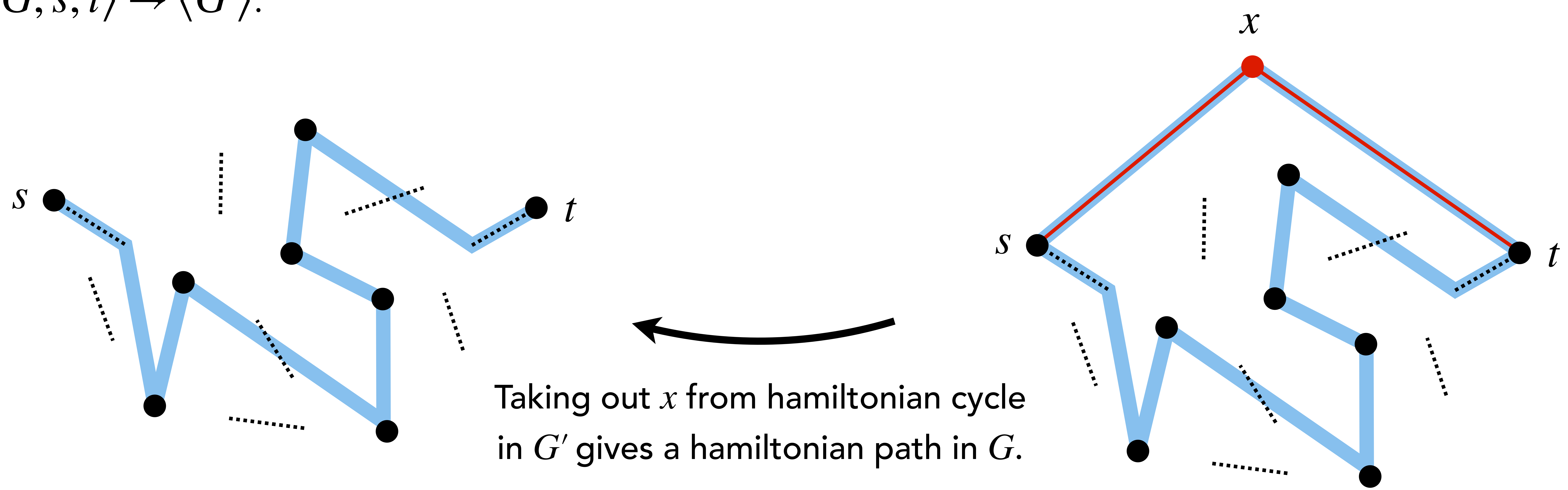
$\langle G, s, t \rangle \to \langle G' \rangle$:



Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:



Taking out x from hamiltonian cycle in G' gives a hamiltonian path in G.

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \to \langle G'\rangle$:

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:

- $G' = G$.

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:

- $G' = G$.
- Add a new vertex x and edges $\{s, x\}$ and $\{t, x\}$.

Hampath $\leq_p$ *Hamcycle*

- *Hampath* $= \{\langle G, s, t\rangle \mid G$ is an undirected graph with a hamiltonian path from s to $t\}$
- *Hamcycle* $= \{\langle G'\rangle \mid G'$ is an undirected graph with a hamiltonian cycle$\}$

$\langle G, s, t\rangle \rightarrow \langle G'\rangle$:

- $G' = G$.
- Add a new vertex x and edges $\{s, x\}$ and $\{t, x\}$.

Correctness is easy.